

# Rotation Response of a Rigid Body under Seismic Excitation

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## Summary

After the 1994 Sanriku-Haruka-Oki, Japan, earthquake, rotation of tombstones along the vertical axis occurred in a graveyard about 34 m from the JMA Hachinohe Observatory where strong motion was recorded. The properties of seismic motion that make a rigid rectangular solid body rotate are discussed. Shake table tests are conducted to reproduce the rotation response of Japanese-style tombstones which typically consist of several stone blocks whose shapes are rectangular solids. Data obtained from those tests are used to

calibrate a numerical model by the three-dimensional Distinct Element Method (3D-DEM). Results of the shaking tests and numerical analyses show that rotation of a rigid rectangular solid body may be caused by the combination of the rocking of the body and particle motion of the input acceleration. Rotation behavior of an actual tombstone is simulated based on the observed accelerogram. Findings show that one or two cycles of particle motion near peak acceleration caused the rotation.

## Introduction

Field surveys to count overturned tombstones have been made to assess the local seismic intensity after destructive earthquakes in Japan (e.g., Ikegami and Kishinoue 1947; Mononobe 1926; Omote et al. 1977; US Far East Command 1949). Seismic intensity can be estimated from the ratio of overturned tombstones at a site. Many theoretical studies of the overturn of a rigid body have been conducted in the past; e.g., Housner 1963; Ikegami and Kishinoue 1947; Iyengar and Manohar 1991; Pompei et al. 1998; Scalia and Sumbatyan 1996; Shenton and Jones 1991.

The rotation response of a tombstone along its vertical axis (Figure 1) during an earthquake has been reported and photographed for many years as an evidence of occurrence. For example, the 1961 Kita Mito earthquake (Kishinouye and Onda 1961), the 1964 Niigata earthquake (Kawasumi 1968), the 1968 Tokachi-Oki earthquake (Matsuda 1968), the 1993 Kushiro-Oki earthquake (Tobishima Corp. 1993), and the 1993 Hokkaido-Nansei-Oki earthquake (Sato Kogyo Corp. 1993), and the 1995 Hyogoken-Nambu earthquake (Midorikawa and Fujimoto 1996).

However, major interest of those reports are the number of overturned tombstones. Therefore research dealing with the rotation response of a tombstone and its relation to local site condition and seismic motion has yet to be conducted in detail.

## Shake Table Test

The objective is to reproduce and observe the rotation response of a rigid body and to obtain data for calibration of the numerical model. The shake table used was capable of one-direction shaking.

The rigid body, composed of granite, was placed on a granite plate fixed on the shaking table (Figure 2 and 3). Three accelerometers were installed to measure the input and response at the top of the rigid body. Photographs were taken every 0.2 seconds from above the shake table to record the rotation of the rigid body (Figure 4). Figure 5 shows measured acceleration and rotation angle.

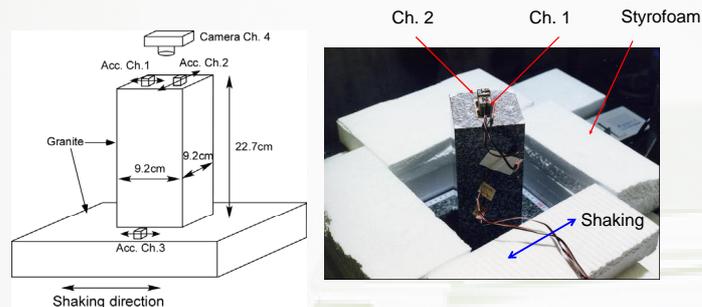


Figure 2 Schematic view of the experiment. 3 acceleration transducer was implemented. Rotation was recorded by photographs.

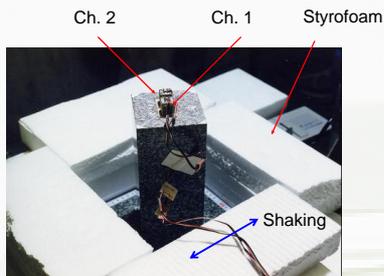


Figure 3 View of the experiment. Styrofoam was used for protection.



Figure 1 An example of rotation of a tombstone. A tombstone consisted of three rectangular granite blocks. Bottom one slid while the middle and top ones rotated together. During the 1994 Sanriku-Haruka-Oki, Japan, earthquake, 80 % of tombstones in a graveyard near the JMA Hachinohe observatory rotated in the same direction without overturning.

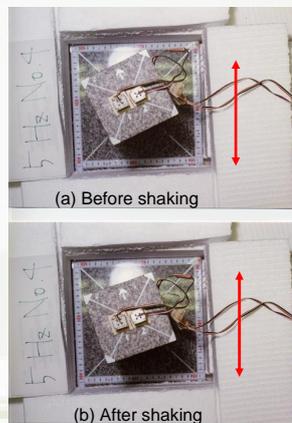


Figure 4 Before (a) and after (b) rotation of a rigid body by sin wave of 5 Hz. Initially a rigid body was placed at some angles against shaking so that it would rotate. In this case, measured rotation angle was about 10 degrees.

Table 1 Properties of the model tombstone

Material	Granite	Units
Dimension	9.2 x 9.2 x 22.7	cm
Density	2.7 x 10 <sup>3</sup>	kg/m <sup>3</sup>
Mass	5.2	kg

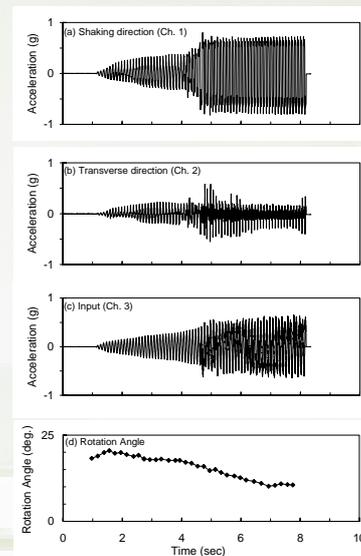


Figure 5 Measured time histories for 10 Hz input: (a) acc. in the shaking direction, (b) acc. in the transverse direction, (c) input acc., and (d) rotation angle. Time history of rotation angle was obtained from photographs taken above the shake table.

## Numerical Analysis

The rigid body was modeled by the 3D-DEM as a single rectangular parallelepiped element with six degrees of freedom (Figure 6). Contact forces are calculated from the linear springs and dash-pots attached to the bottom corners of the rigid body. Although the dynamic motion of a rigid body is a highly nonlinear phenomenon that includes slip, rotation, rocking, jumping, overturning, and combinations of those motions, the DEM is able to simulate such a complicated behavior as a problem of the four contact points. In what follows, initial position and rotation angle are defined as in Figure 7.

Values of the spring and damping constants were determined by fitting the rotation angle of the analysis to the one obtained experimentally (Figure 8). For simplicity, both the horizontal and vertical springs and dash-pots were assumed to have the same properties. Figure 8 and 9 compare results of experiments and simulation.

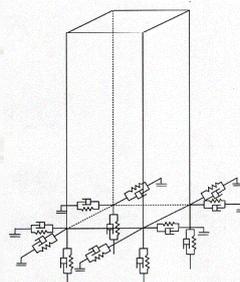


Figure 6 DEM model

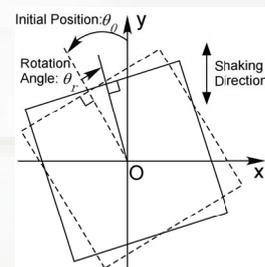
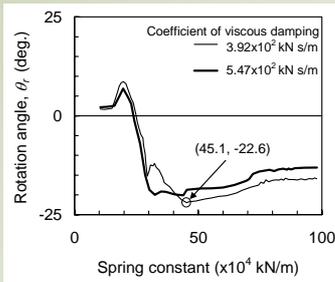
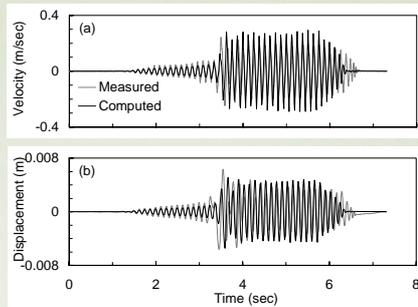


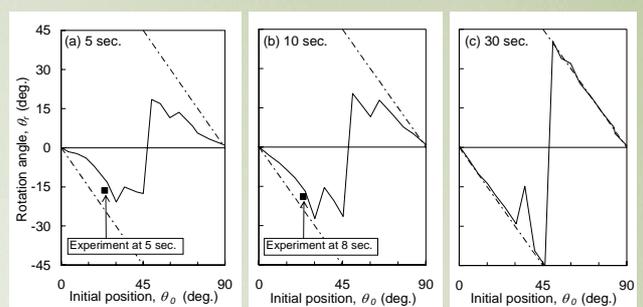
Figure 7 Definitions of the initial and rotation angles.



**Figure 8**  
Rotation angle versus spring constant for the input motion of 20 Hz. Model parameters are determined by fitting the rotation angle as regards the change of spring constant. The computed rotation angle is plotted against spring constant for two different damping constants;  $3.92 \times 10^2$  and  $5.47 \times 10^2$  (kN s/m). The target rotation angle is  $-22.6^\circ$ . As seen in the figure, measured rotation angle is obtained when the spring constant is  $45.1 \times 10^4$  (kN/m) and damping constant of  $3.92 \times 10^2$  (kN s/m).



**Figure 9**  
Measured vs. computed velocity (a) and displacement (b) at the top of the rigid body. Input motion in the tests was interpolated linearly to obtain the time increment  $\Delta t = 10^{-4}$  (s). The same dimensions and mass of the rigid body as the shaking table test were employed. The coefficient of kinetic friction was assumed to be  $\approx 0.56$ ; 80% of the static friction coefficient of granite, approximately 0.7 (Stesky et al. 1974).



**Figure 10**  
Computed rotation angle versus the initial position for shaking durations of (a) 5, (b) 10, and (c) 30 s under one directional shaking. The input acceleration is sinusoidal with a frequency of 10 Hz and maximum amplitude of 0.46g. As a comparison, experimental results of the input acceleration amplitude 0.65 g and frequency 10 Hz is plotted with solid square. The shaking duration of the experiment was about 8 seconds. Computed results show that no rotation occurred at an initial angle of  $0^\circ$  or  $90^\circ$  because the initial position is axisymmetric to the direction of shaking. For the other angles, the rigid body rotated until a side became parallel or perpendicular to the shaking direction. Dashed lines in figures denote the maximum rotation angle at a given initial angle for infinite shaking. Figures show that the rotation angle of the rigid body depends on the initial angle and duration of shaking.

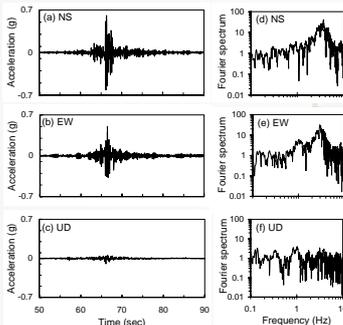
## Observed Tombstone Rotation and Numerical Simulation

The Sanriku-Haruka-Oki earthquake occurred on December 28, 1994, at 9:19 p.m. local time (Kabayama et al. 1995). The epicenter ( $N40.45^\circ, E143.72^\circ$ ) was located about 180 km east of Hachinohe City. There the magnitude was 7.5 and the seismic intensity 6 on the JMA (Japan Meteorological Agency) seismic intensity scale. Figure 11(a)-(c) show the time histories of acceleration. Figure 11(d)-(f) show the Fourier spectra of the records. The predominant frequencies of each component are 4 Hz for the NS, 3 Hz for the EW, and 1 Hz for the vertical one.

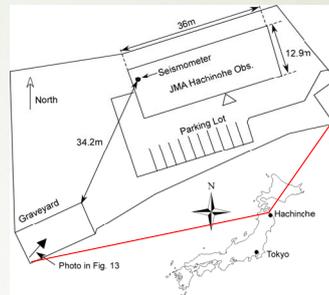
Tombstone rotation was present after the earthquake in a graveyard next to the JMA Hachinohe Observatory where strong motion was recorded (Figure 12). The observatory is only 34 m from the graveyard. Both are on a small hill with an elevation of 27 m (Figure 13). The top of this hill is covered by 8 to 9 m of weathered soil underlain by bedrock. Most of the tombstones faced about  $N25^\circ W$  (Figure 14).

25 tombstones out of 30 have rotated similarly in the counterclockwise direction. Even flat, panel-shaped tombstones have rotated but remain standing. Rotation has occurred for both the top and underlying stones, except six near the south corner of the cemetery where there was a 1.5 m cut (free face) just behind them. They have been moved forward, opposite to the direction of the cut.

The position of the tombstone used in the simulation is shown as a solid square in Fig. 14. This tombstone is located away from the free face and it is relatively a small one, consisting of 2 stone blocks (Figure 15). Only the top one was used in the simulation as the lower block did not move. The observed rotation angle of this tombstone was about  $16.7^\circ$  counterclockwise. Figure 16 explores the cause of rotation.



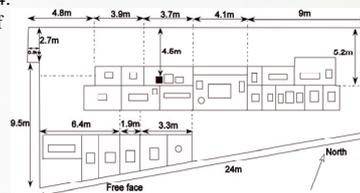
**Figure 11**  
Measured time histories of acceleration and Fourier spectra (a) NS, (b) EW, and (c) UD component during the 1994 Sanriku-Haruka-Oki, Japan, earthquake recorded at the JMA Hachinohe Observatory.



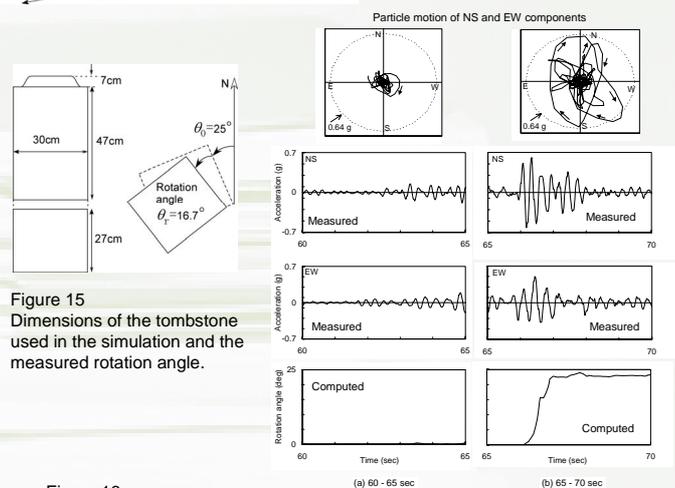
**Figure 12**  
Location of the JMA (Japan Meteorological Agency) Hachinohe Observatory.



**Figure 13**  
View of the graveyard (near the JMA Hachinohe Obs.) where rotation of tombstones occurred.



**Figure 14**  
Disposition of tombstones in the graveyard close to the JMA Hachinohe Obs. The tombstone shown by the filled square was used in the numerical analysis.



**Figure 15**  
Dimensions of the tombstone used in the simulation and the measured rotation angle.

**Figure 16**  
Particle motion and the time histories of measured accelerations (NS and EW components) and simulated time histories of the rotation angle: (a) 60-65 sec, (b) 65-70 sec. Simulation of tombstone rotation during the earthquake was made with the 3 components of strong motion shown in Fig. 11. To clarify the timing of rotation, segments of (a) 60-65 seconds, just before the peak acceleration, and (b) 65-70 seconds, which includes the peak acceleration were input. The radius of the dashed circle in the particle motion plots corresponds to 0.64g; the acceleration required to initiate rocking of the tombstone. Particle motion is nearly circular with a radius of about 0.3g. The rotation angle in the simulation is small,  $0.4^\circ$ . Particle motion forms a large ellipse and has a maximum acceleration of 0.69g in the  $S45^\circ E$  direction. The large locus begins at the center and moves north clockwise (arrows in the figure). As shown, the tombstone rotates about  $23^\circ$  counterclockwise; the reverse of particle motion.

## Conclusions

We conducted shake table tests and numerical studies to investigate the properties of the strong motion which caused Japanese tombstones to rotate along the vertical axis during the 1994 Sanriku-Haruka-Oki, Japan, earthquake. The numerical model was calibrated and validated by data obtained from shaking table tests. Simulation showed that elliptical particle motion at about peak input acceleration made tombstones rotate. Relationships found between the properties of ground motion and the rotation of a rigid body are (1) A rigid body may be rotated by one-direction seismic ground motion that has a predominant frequency as low as the rocking frequency at the maximum rocking amplitude and an acceleration amplitude larger than the one required to initiate rocking. Except in rare cases as in (1), (2) rotation of a rigid body is caused by the elliptical or circular particle motion of strong ground motion. The direction of rotation of a rigid body is opposite to the direction of the particle motion of input acceleration due to inertial force acting on the body. Rotation of a rigid body therefore may be a consequence of rocking and the particle motion of input acceleration.