Amplification of Ground Motion Rotations by Local Geology

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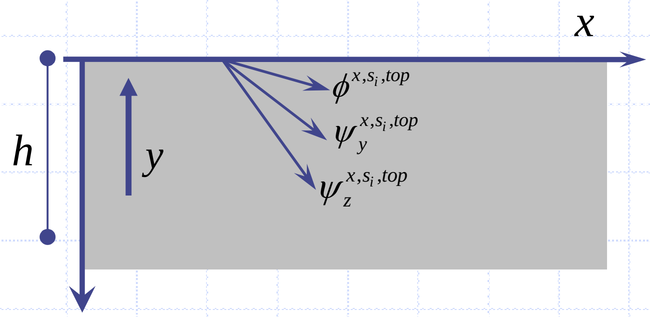
Motivation

- **Site effects** influence the nature of ground motion, causing marked amplification phenomena .
- Many previous works deal with motion amplification, while **little was done to characterize rotations**.
- **Scarce records** exist of spatial gradients of motion.
- **Mathematical models** may be used to mitigate the lack of field information.
- **Meshless methods** seem to be adequate to study these phenomena.

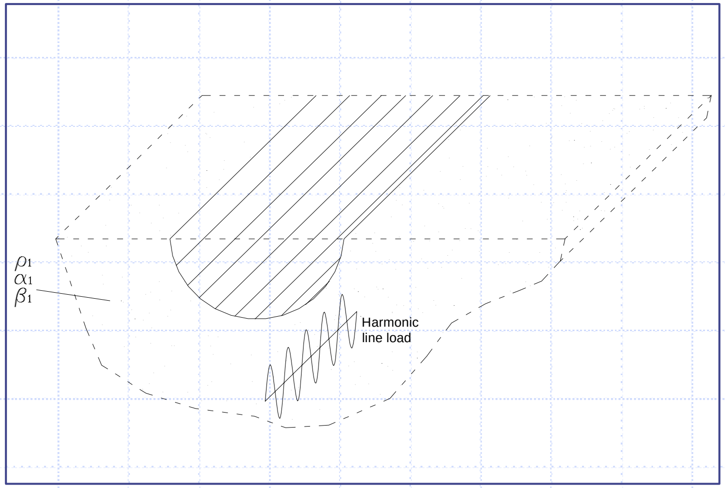
Objectives

- Implement a **meshless** technique for simulating wave propagation in elastic media, making use of **fundamental solutions** that take into account the free surface of half-spaces.



- Study the rotations generated by **seismic wave** propagation in a **half-space**.

- Understand the effect of simple **local topography** on the rotations generated by seismic waves.



Presentation Outline

Mathematical formulation of the half-space model.

Formulation of the MFS.

Model verification.

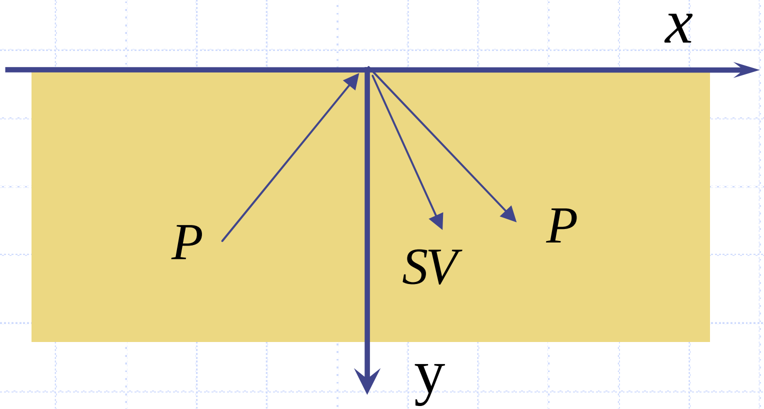
Numerical simulations:

- motions and rotations in a half-space
- motions and rotations in the presence of a shallow valley

Conclusions.

Rotations in a Half-Space under incidence of Plane Waves

HALF SPACE



$$k_n = \frac{\omega}{c} \quad k_\alpha = \sqrt{\frac{\omega^2}{\alpha^2} - k_n^{\,2}} \quad k_\beta = \sqrt{\frac{\omega^2}{\beta^2} - k_n^{\,2}}$$

Displacement potentials:

$$\Phi = \left(A_1 e^{ik_\alpha y} + A_2 e^{-ik_\alpha y}\right) \cdot e^{-ik_n x}$$

$$\Psi = \left(B_1 e^{ik_\beta y} + B_2 e^{-ik_\beta y}\right) \cdot e^{-ik_n x}$$

Displacements :

$$u_x = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial y} \qquad u_y = \frac{\partial \Phi}{\partial y} + \frac{\partial \Psi}{\partial x}$$

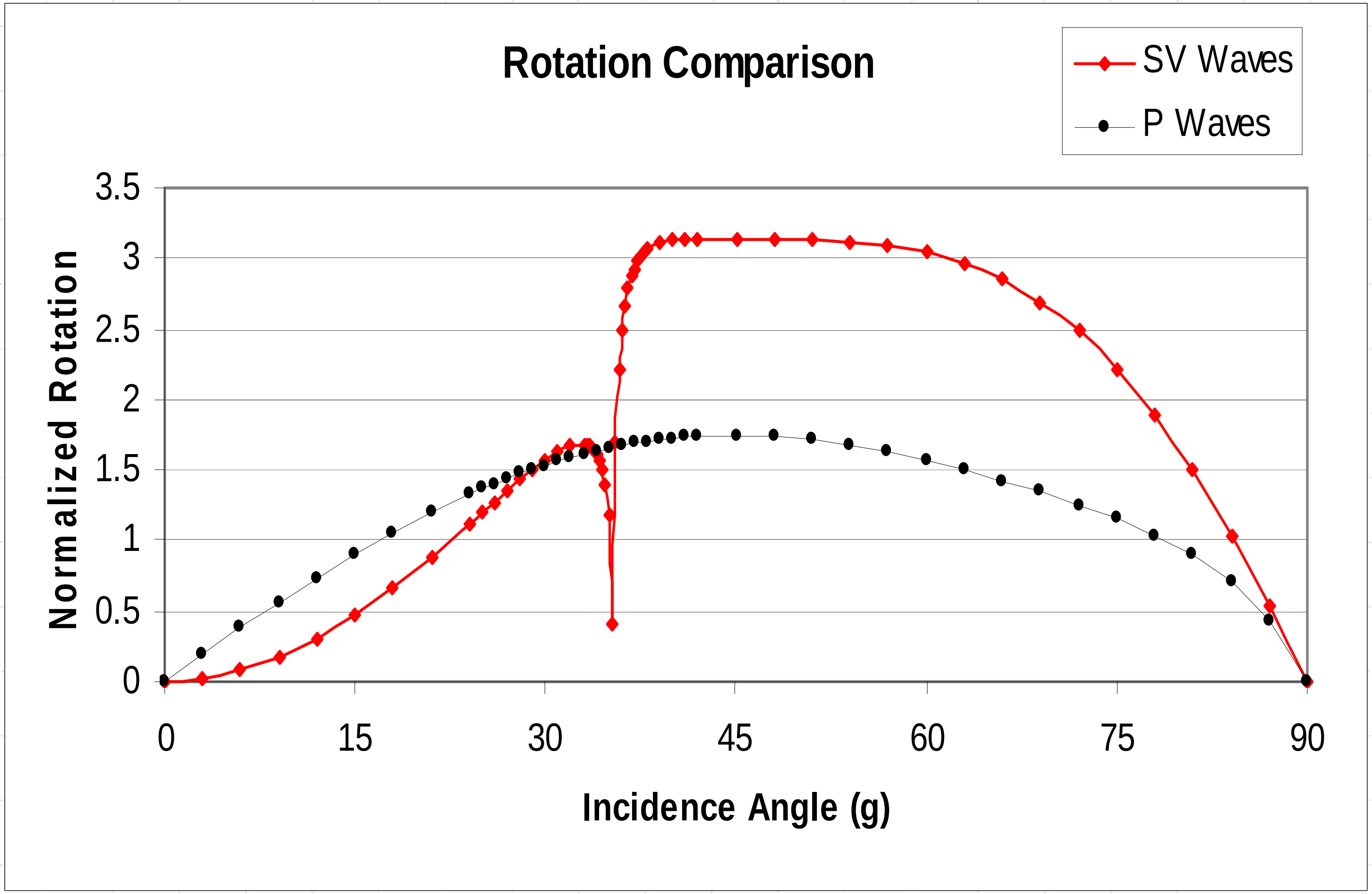
Rotations:

$$\phi = \frac{1}{2} \nabla \times \mathbf{u} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x}\right)$$

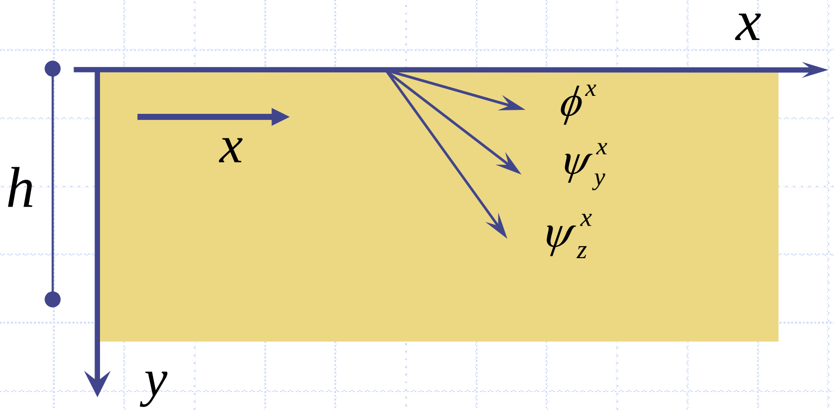
$$\phi = \frac{1}{2}\left[\left(B_1 + B_2\right)\left(k_\beta^{\,2} + k_n^{\,2}\right)\right] \cdot e^{-i\,k_n x}$$

Rotations in a Half-Space under incidence of Plane Waves

HALF SPACE – Rotations?



2.5D HALF SPACE



$$k_p = \omega/\alpha \qquad k_s = \omega/\beta$$

$$E_A = \frac{1}{2\rho\,\omega^2 L_x}$$

$$E_{b0} = \mathrm{e}^{-\mathrm{i}\nu_n|y|}$$

$$E_{c0} = \mathrm{e}^{-\mathrm{i}\gamma_n|y|}$$

$$\nu_n = \sqrt{(k_p)^2 - k_z{}^2 - k_n{}^2} \qquad \gamma_n = \sqrt{(k_s)^2 - k_z{}^2 - k_n{}^2}$$

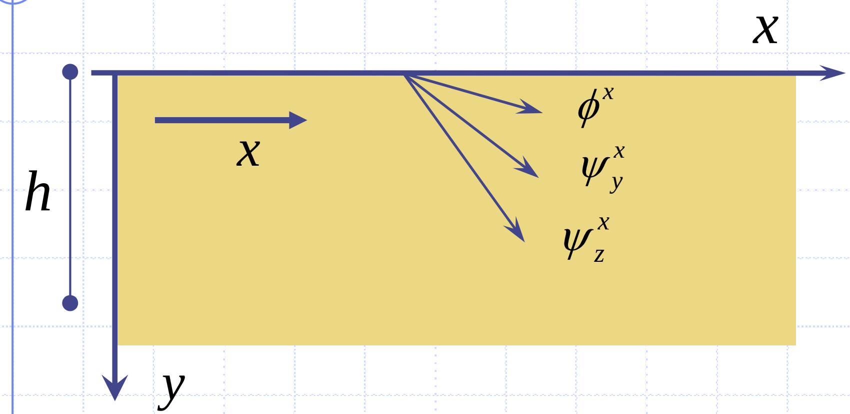
Displacement potentials:

$$\phi_0^x = E_a \sum_{n=-N}^{n=+N} \left(\frac{k_n}{\nu_n} E_{b0} A_n^x \right) E_d$$

$$\psi_{y0}^x = E_a k_z \sum_{n=-N}^{n=+N} \left(\frac{E_{c0}}{\gamma_n} B_n^x \right) E_d$$

$$\psi_{z0}^x = -E_a \sum_{n=-N}^{n=+N} \left(E_{c0} C_n^x \right) E_d$$

2.5D HALF SPACE



$$k_p = \omega / \alpha \qquad k_s = \omega / \beta$$

$$E_A = \frac{1}{2\rho\, \omega^2 L_x}$$

$$E_{b0} = \mathrm{e}^{-\mathrm{i}\nu_n |y|}$$

$$E_{c0} = \mathrm{e}^{-\mathrm{i}\gamma_n |y|}$$

$$\nu_n = \sqrt{\left(k_p\right)^2 - k_z^{\,2} - k_n^{\,2}} \qquad \gamma_n = \sqrt{\left(k_s\right)^2 - k_z^{\,2} - k_n^{\,2}}$$

Fundamental solution:

$$G_{xx}(x_0, y_0, x, y) = G_{xx}^{full}(x_0, y_0, x, y) + E_a \sum_{n=-N}^{n=+N} \begin{pmatrix} A_n^x \dfrac{-\mathrm{i}k_n^{\,2}}{\nu_n} E_{b0} - \\ -\mathrm{i}\gamma_n E_{c0} C_n^x - \dfrac{\mathrm{i}k_z^{\,2}}{\gamma_n} E_{c0} B_n^x \end{pmatrix} E_d$$

$$G_{yx}(x_0, y_0, x, y) = G_{yx}^{full}(x_0, y_0, x, y) + E_a \sum_{n=-N}^{n=+N} \begin{pmatrix} -\mathrm{i}k_n A_n^x E_{b0} + \\ +\mathrm{i}k_n C_n^x E_{c0} \end{pmatrix} E_d$$

$$G_{zx}(x_0, y_0, x, y) = G_{zx}^{full}(x_0, y_0, x, y) + E_a \sum_{n=-N}^{n=+N} \begin{pmatrix} \dfrac{-\mathrm{i}k_z k_n}{\nu_n} A_n^x E_{b0} + \\ + \dfrac{\mathrm{i}k_z k_n}{\gamma_n} B_n^x E_{c0} \end{pmatrix} E_d$$

2.5D HALF SPACE – Rotations?

$$\phi = 0.5\left(\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x}\right)$$

Horizontal load

$$\phi^x = -0.5\, E_a \sum_{n=-N}^{N} E_d E_{c0} \begin{pmatrix} B_n^x k_z^2 + \\ C_n^x(\gamma_n^2 + k_n^2) \end{pmatrix} + \phi_{full}^x$$

Vertical load

$$\phi^y = 0.5\, E_a \sum_{n=-N}^{N} \frac{E_d E_{c0}}{\gamma_n} \begin{pmatrix} C_n^y k_z^2 + \\ B_n^y(\gamma_n^2 + k_n^2) \end{pmatrix} + \phi_{full}^y$$

In 2D problems, with k_z=0:

$$\phi^x = -0.5\, E_a \sum_{n=-N}^{N} E_d E_{c0} \left(C_n^x(\gamma_n^2 + k_n^2)\right) + \phi_{full}^x$$

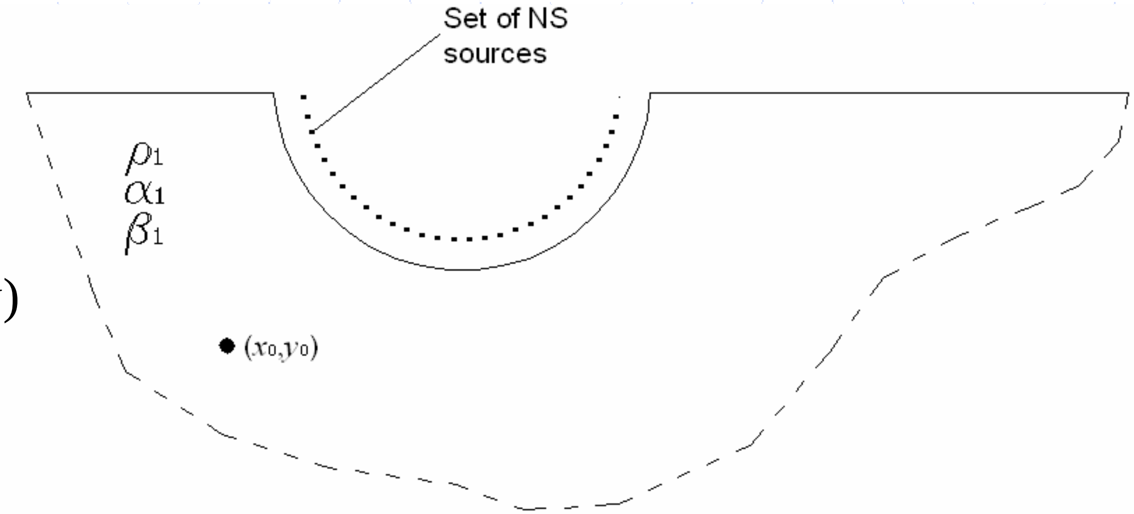
In 2D problems, with k_z=0:

$$\phi^y = 0.5\, E_a \sum_{n=-N}^{N} \frac{E_d E_{c0}}{\gamma_n} \left(B_n^y(\gamma_n^2 + k_n^2)\right) + \phi_{full}^y$$

(M)ethod of (F)undamental (S)olutions

Displacements in the host medium:

$$u_i(x,y)=\sum_{n=1}^{NS}\sum_{j=1}^{3}Q_{nj}G_{ij}(x_n,y_n,x,y)$$



where $G_{ij}^{(m)}(x_n^{(m)},y_n^{(m)},x,y)$ is the displacement at point (x,y), in medium *m*, along direction *i*, generated by a load acting along *j* at position $(x^{(m)},y^{(m)})$

Final system of 3xNS *equations on* 3xNS *unknowns*

Time Domain Signals

- The calculations are first preformed in the **frequency domain**.

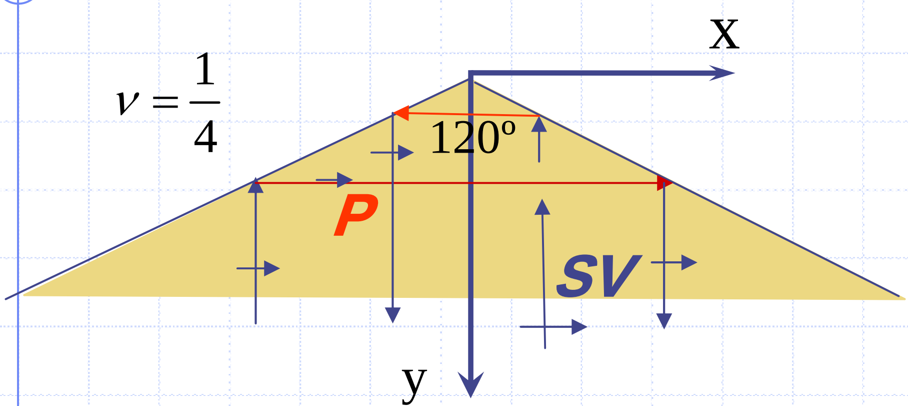
- The time responses can then be obtained by applying an **inverse Fourier transform**.

- The source is assumed to emit a **Ricker pulse**: $u(\tau) = A(1-2\tau^2)e^{-\tau^2}$

- Complex frequencies are used ($\omega_c = \omega - i\eta$, with $\eta = 0.7\Delta\omega$) to prevent the "aliasing" phenomenon.

Method verification

120º INFINITE WEDGE



$$k_\alpha = \frac{\omega}{\alpha} \qquad k_\beta = \frac{\omega}{\beta}$$

Incidence of SV waves

This exact solution can be achieved with vertical propagation of S waves and horizontal propagation of P waves

Displacements :

$$u_x = 2u_0 \cos(k_\alpha x) + 2u_0 \cos(k_\beta y)$$

$$u_y = 0$$

Surface displacements ($y = \frac{|x|}{\sqrt{3}}$):

$$u_x = 4u_0 \cos(k_\alpha x)$$

$$u_y = 0$$

Rotation:

$$\Phi = k_\beta \sin(k_\beta y) = k_\beta \sin(k_\alpha |x|)$$

Normalized Rotation:

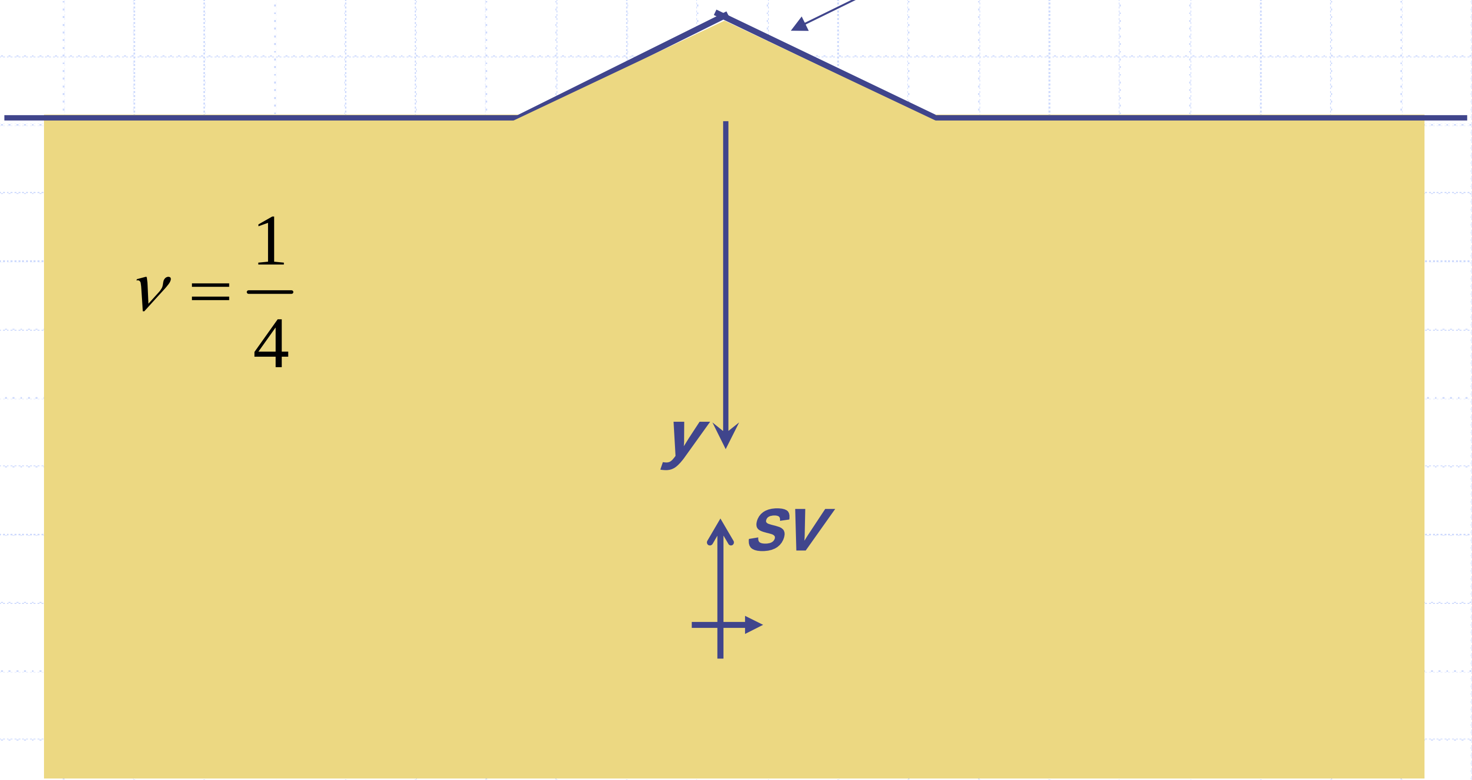
$$\frac{\Phi a}{\eta u_0} = \pi \sin(k_\alpha x)$$

$$\eta = \frac{\omega\, a}{\pi\, \beta}$$

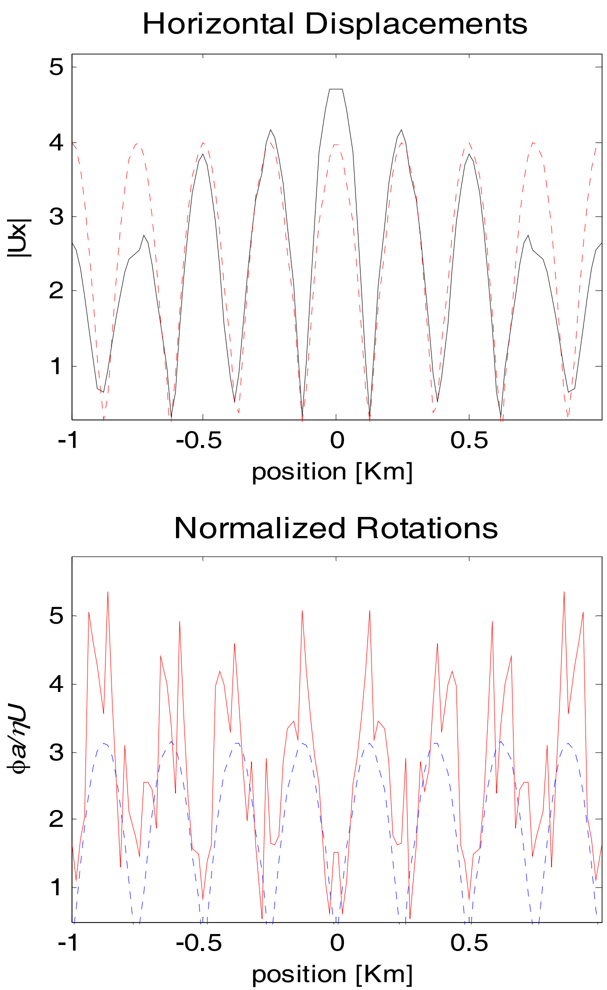
Verification

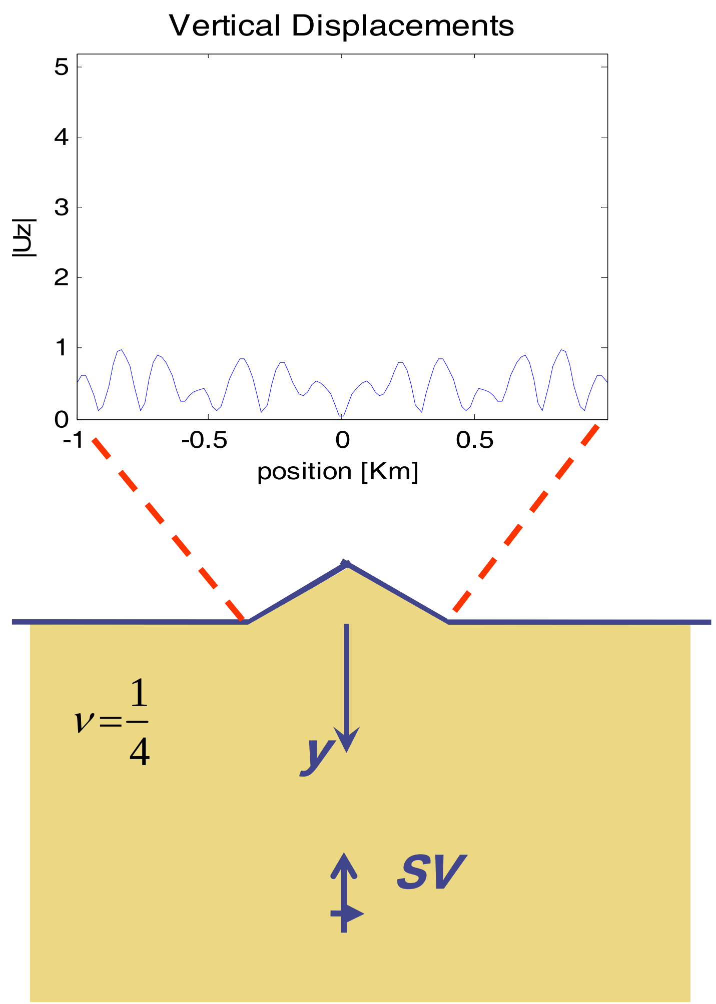
Topography: a 120º at the halfspace surface

Receivers from x=-1000m to x=+1000m



Verification





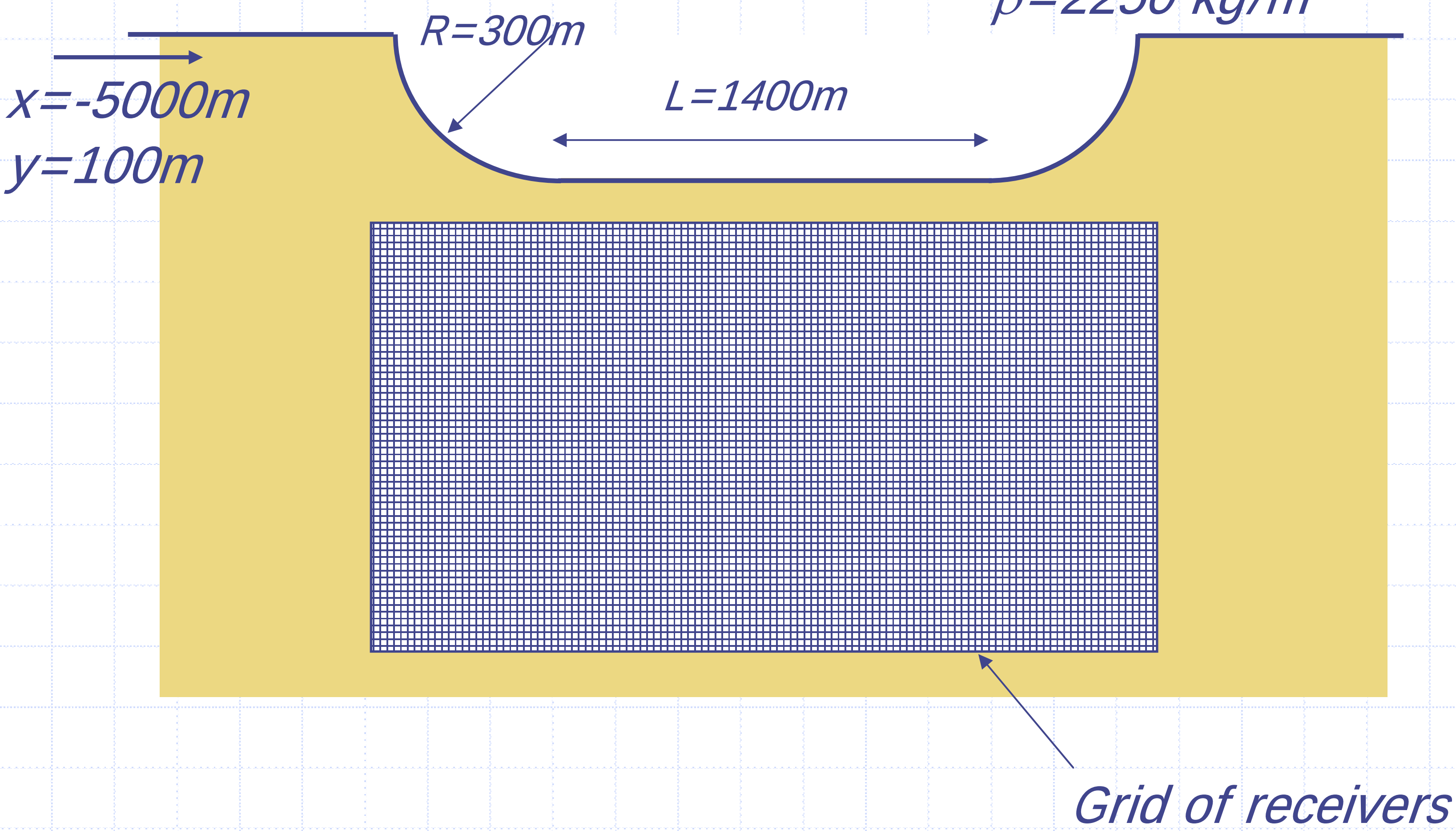
Verification against spatial finite differences

Geometry

$\alpha = 2629$ m/s

$\beta = 1415$ m/s

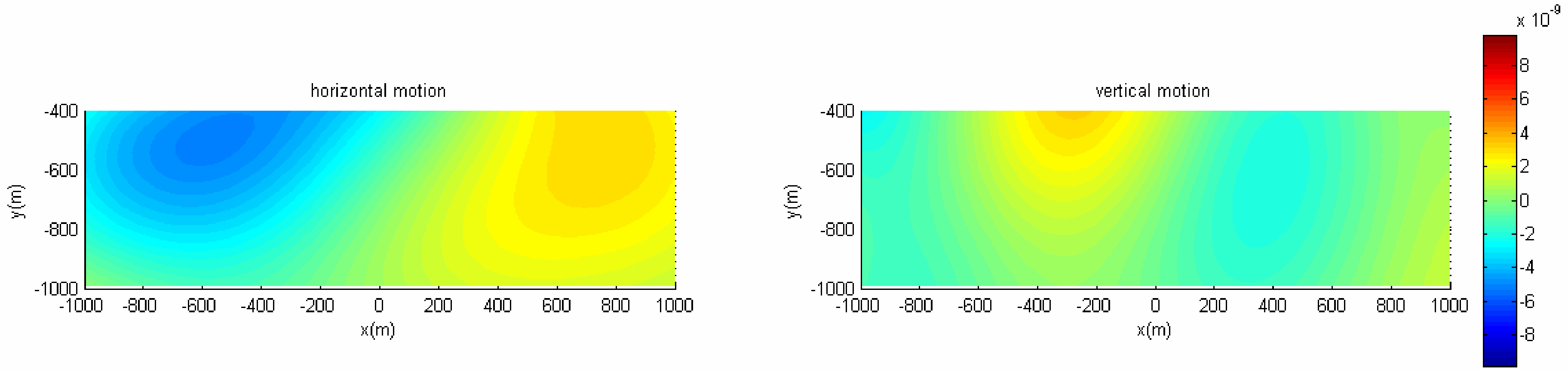
$\rho = 2250$ kg/m^3

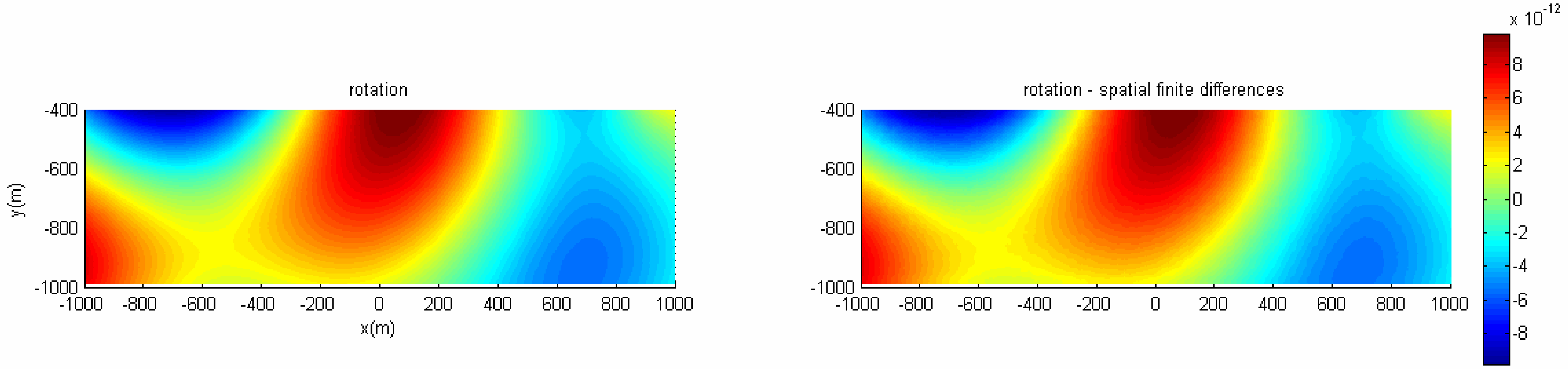


Verification against spatial finite differences

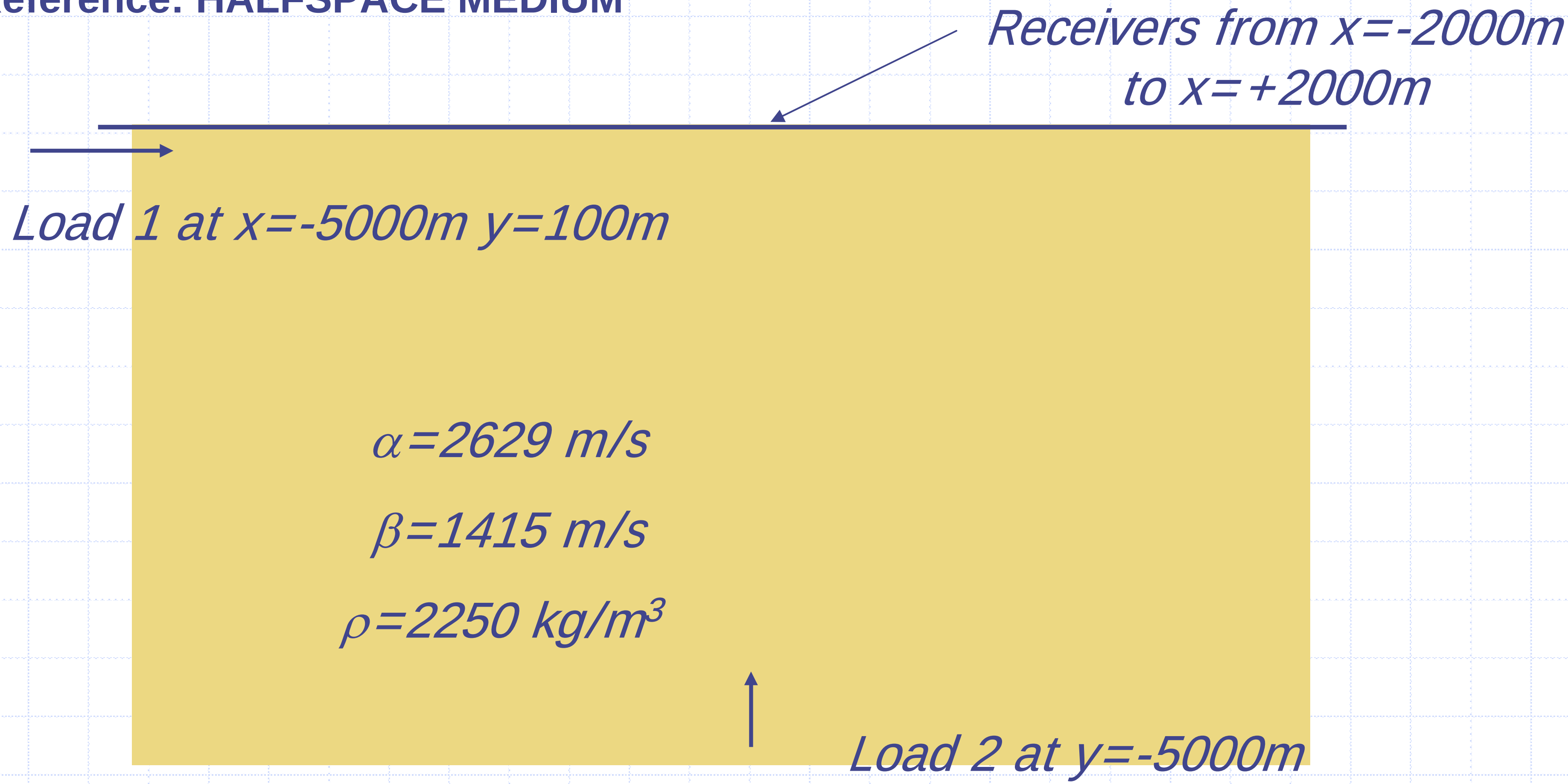
f=1 Hz

$$\phi = 0.5\left(\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x}\right)$$

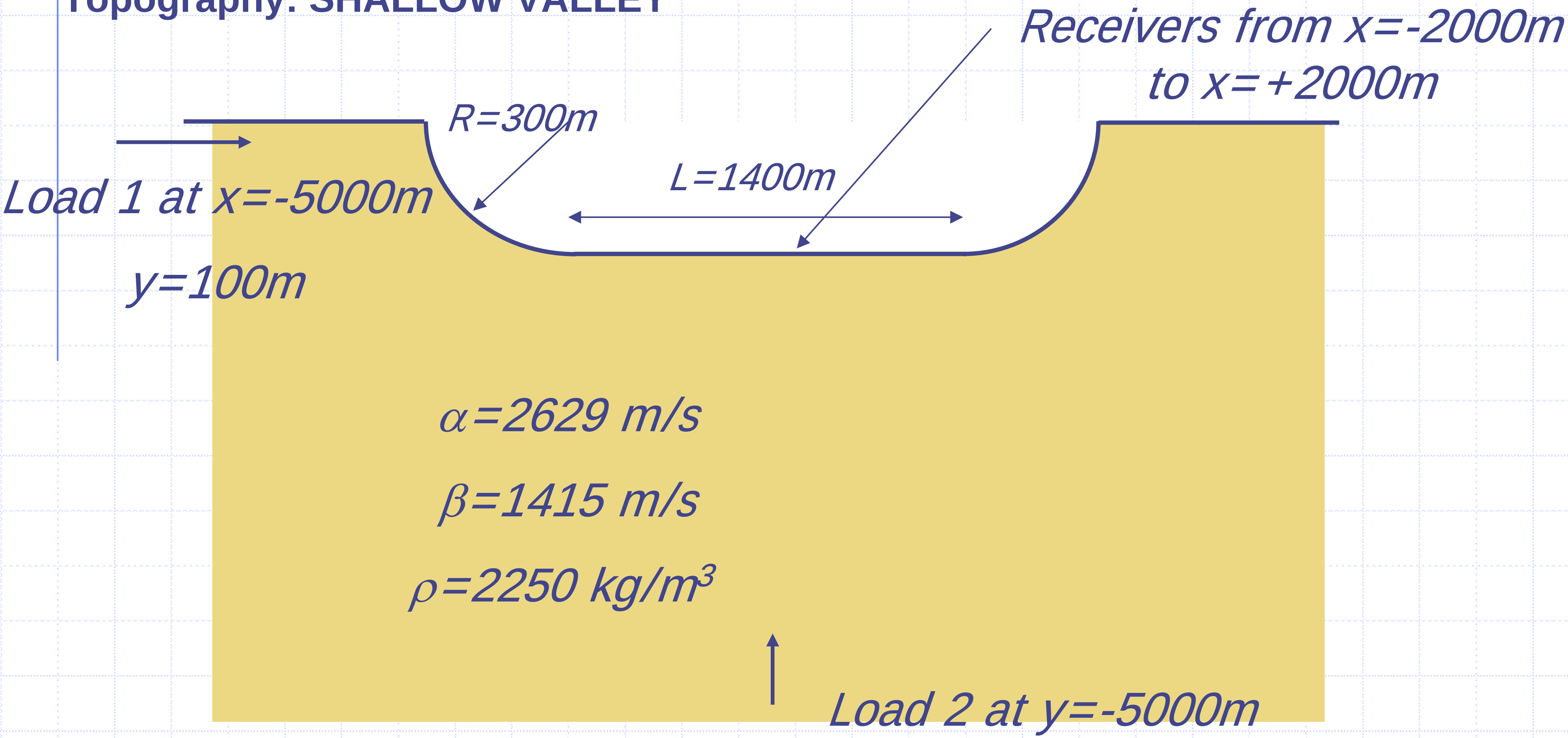




Reference: HALFSPACE MEDIUM



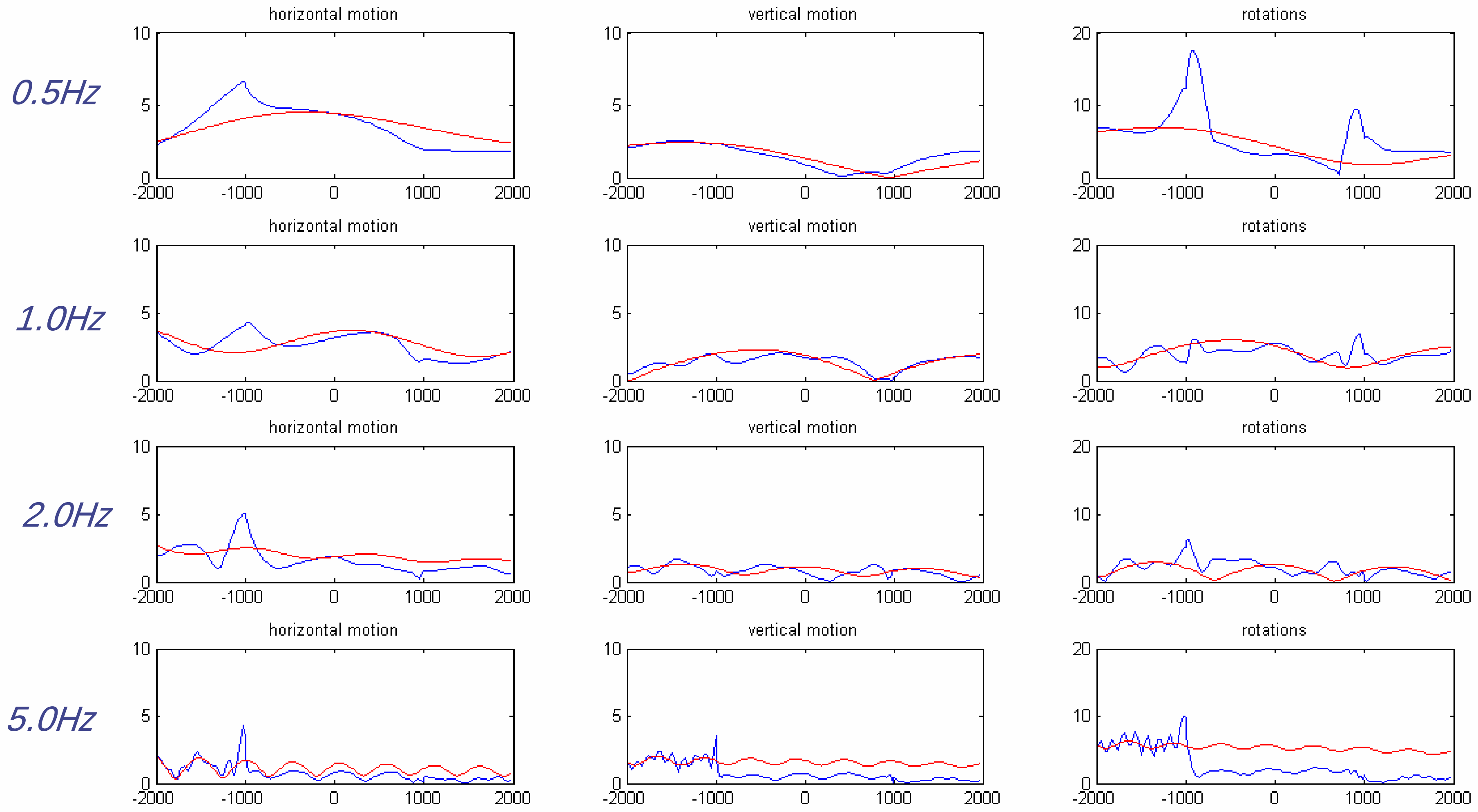
Topography: SHALLOW VALLEY



Numerical Results

CASE 1: Horizontal load (load 1) near the surface
response normalized to the amplitude of the direct incident field (u_0)

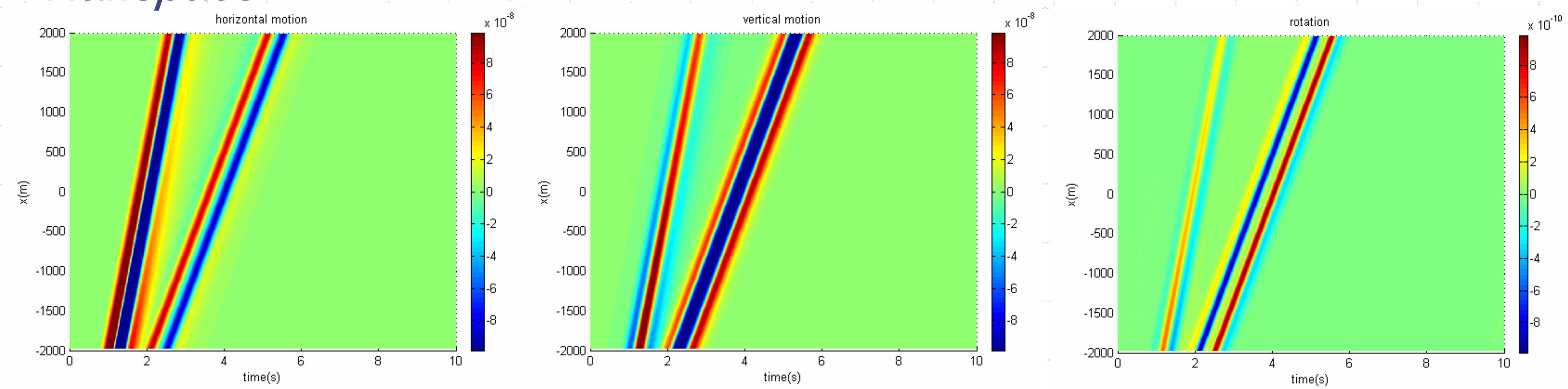
Shallow valley — Halfspace



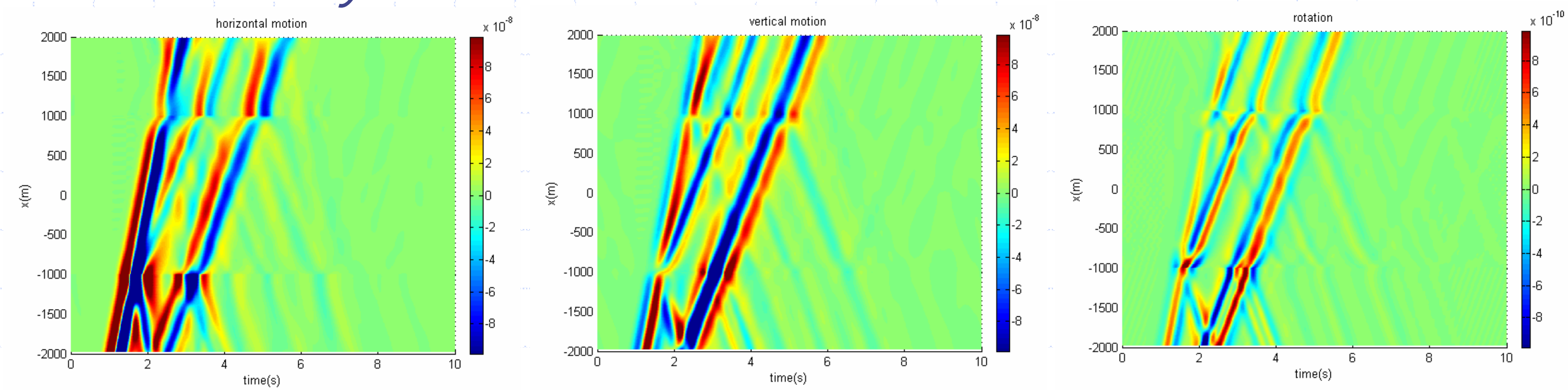
Numerical Results

CASE 1: Horizontal load (load 1) near the surface time response for a central frequency of 1.8 Hz

Halfspace



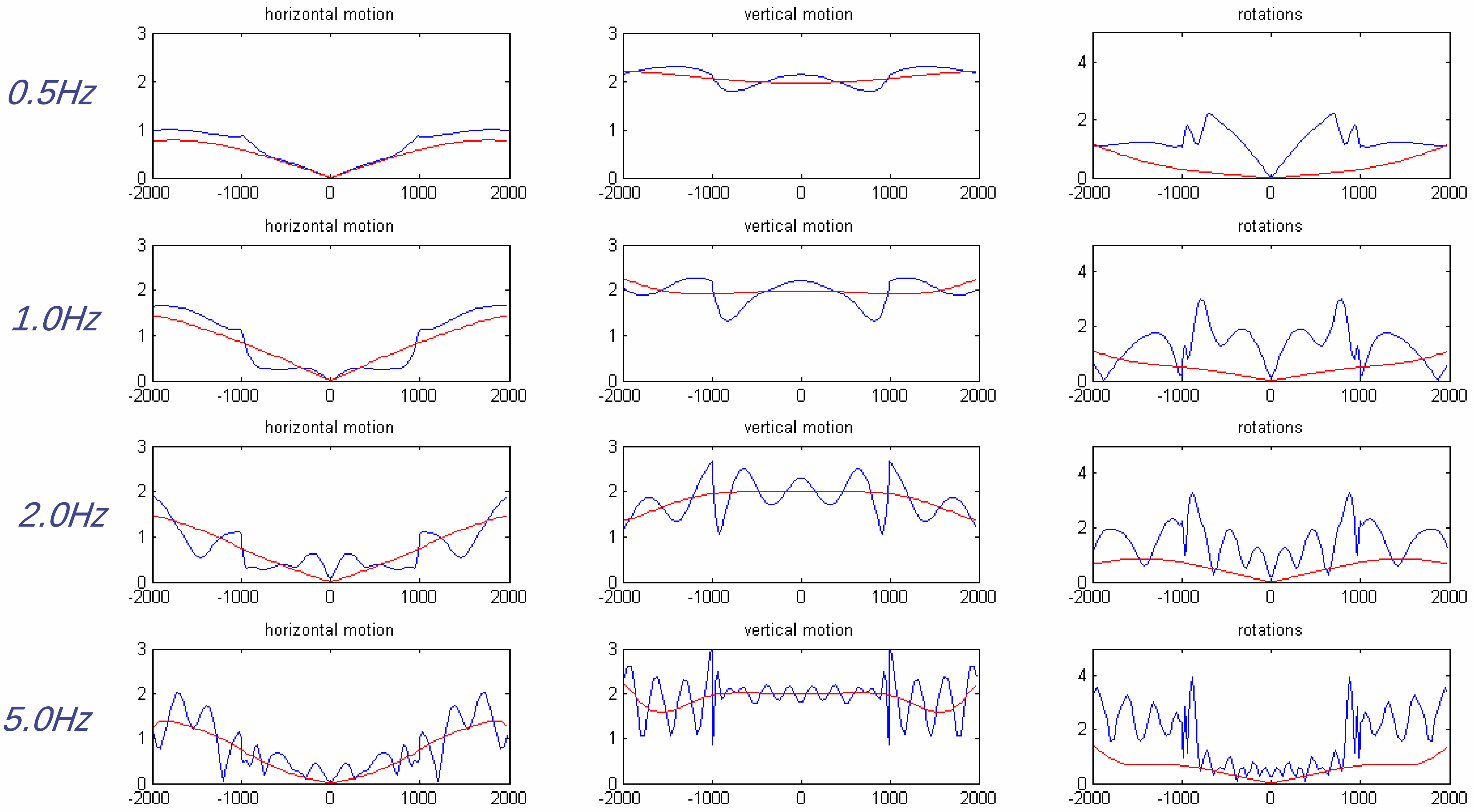
Shallow valley



Numerical Results

CASE 2: Vertical load (load 2) at the center
response normalized to the amplitude of the direct incident field (u_0)

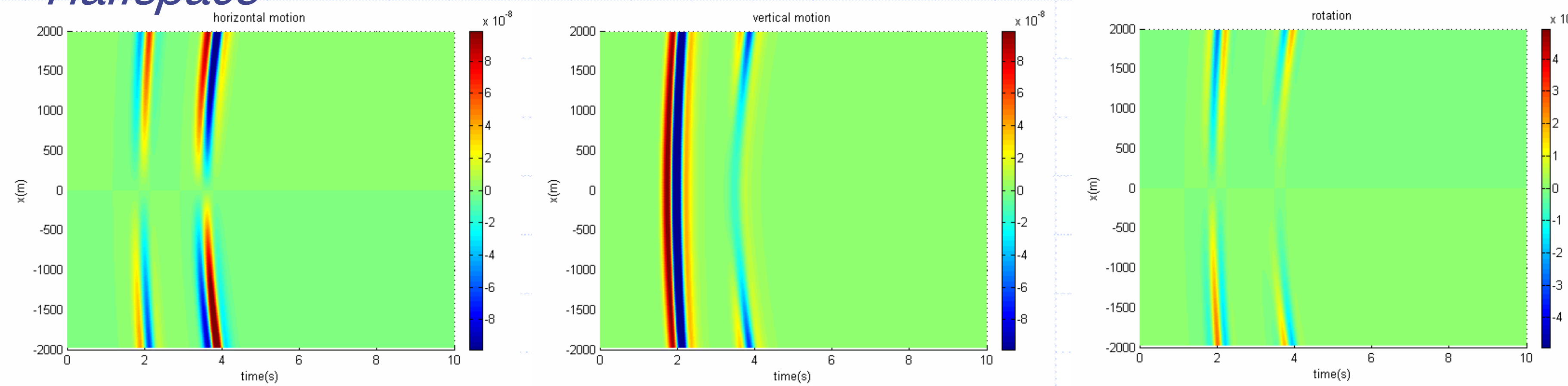
Shallow valley — Halfspace



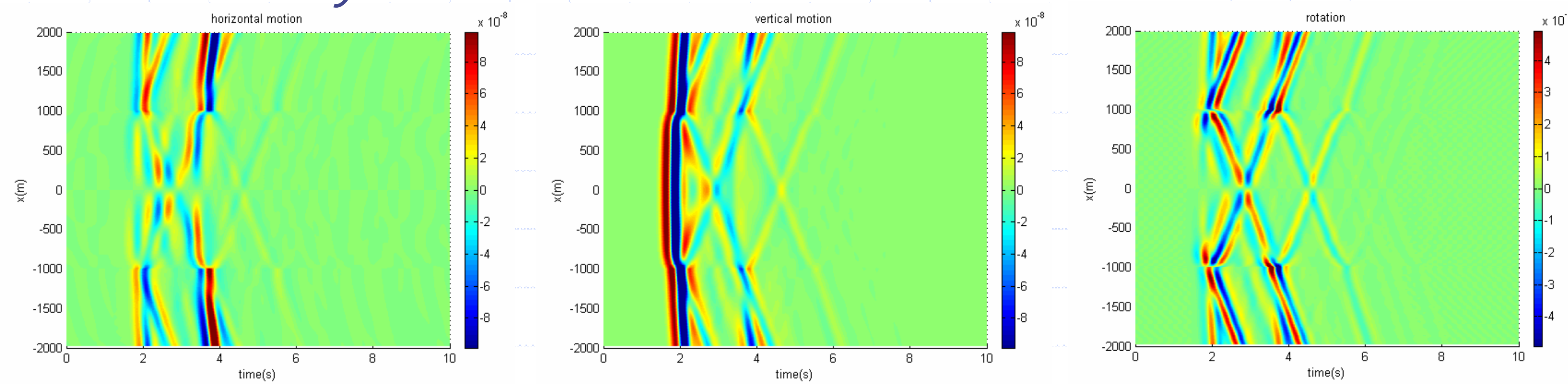
Numerical Results

CASE 2: Vertical load (load 2) at the center time response for a central frequency of 1.8 Hz

Halfspace



Shallow valley



Final remarks

- The present work evaluated the rotational motion originated by line loads in a 2D geometry using the MFS.
- Surface motion, including displacements and rotations, has been analyzed in the presence of an halfspace and of a shallow valley.
- When the system is illuminated by a horizontal load near the surface:
    - -In low frequencies, the rotational motion results reveals stronger amplifications than those provided by the displacement field, particularly in the vicinity of the edges of the valley;
    - -At higher frequencies, both displacements and rotations suffer deamplification for receivers placed at the valley and further away.

Final remarks

When the system is illuminated by a vertical load centered with the topography:

-Globally, the configuration of the response becomes more irregular with the presence of the valley;

-Amplification of rotational motion is much more significant than that of the displacements, both at higher and lower frequencies;

-Very strong amplifications are registered at the horizontal surfaces outside the valley.

For all cases, time responses reveal that the generation of surface and shear waves is the dominant factor for the rotational motion.