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POLITECNICO DI MILANO

Free field rotations: Relevance on buildings in near field

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Free field rotations: relevance on buildings in near field

- Introduction
- Measure of rotations
- Rotational accelerograms
- A civil engineering application
- Conclusions

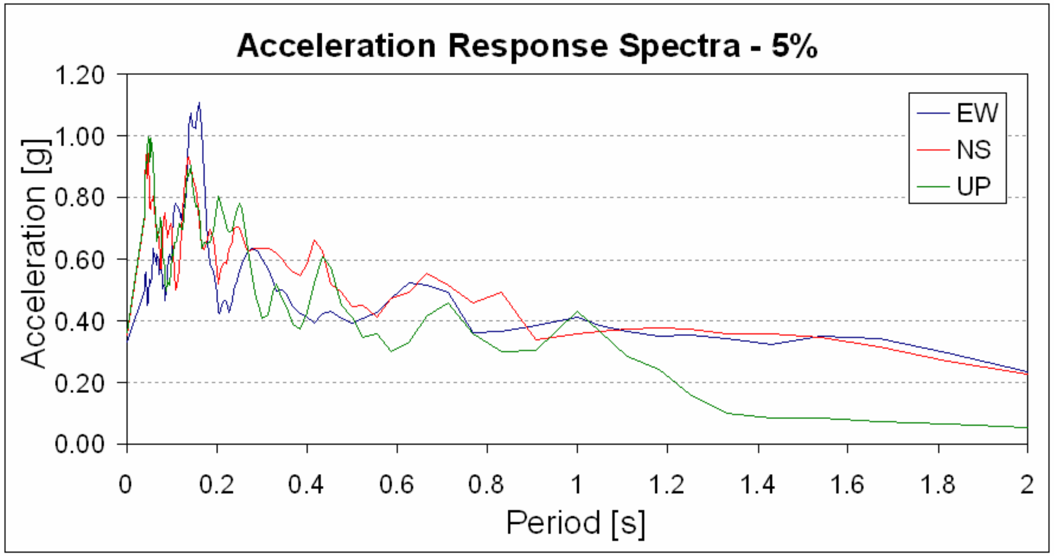
Focus on

• Soil rotation around horizontal axes, during an earthquake

• Evaluation of rotation accelerations

• Relevance of this input motion for relatively tall structures, with reference to the horizontal motion

- **Coherency** model
- **Relative importance** of the vertical to the horizontal input motion



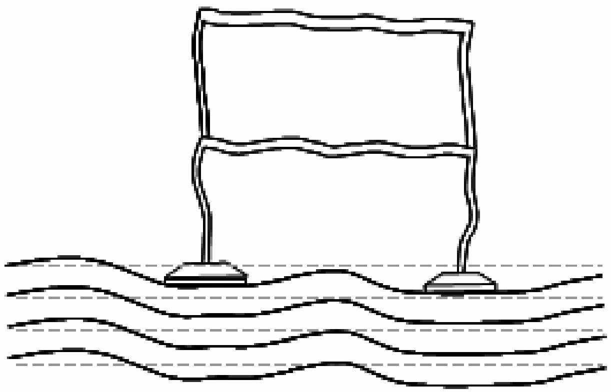
Comparison between the Response Spectrum of vertical motion and that of horizontal motions (EW and NS direction).

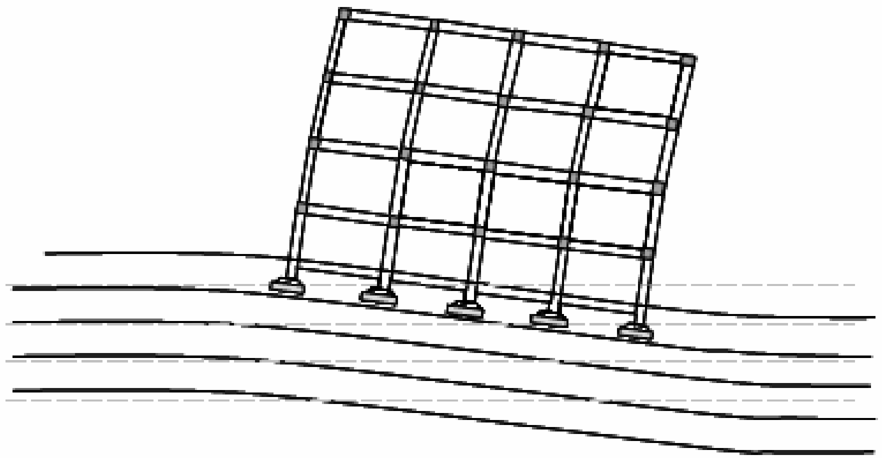
Earthquake event of L'Aquila, Italy (April 6th, 2009)

Focus on

- Effects of rotational motion on engineering structures

High frequency content	Low frequency content





- ➢ Local vibration of beams and columns
- ➢ Meaningless motion of the building center of mass

- ➢ Overturning moment
- ➢ Horizontal displacement of the center of mass
- ➢ **Higher stress in structural element**

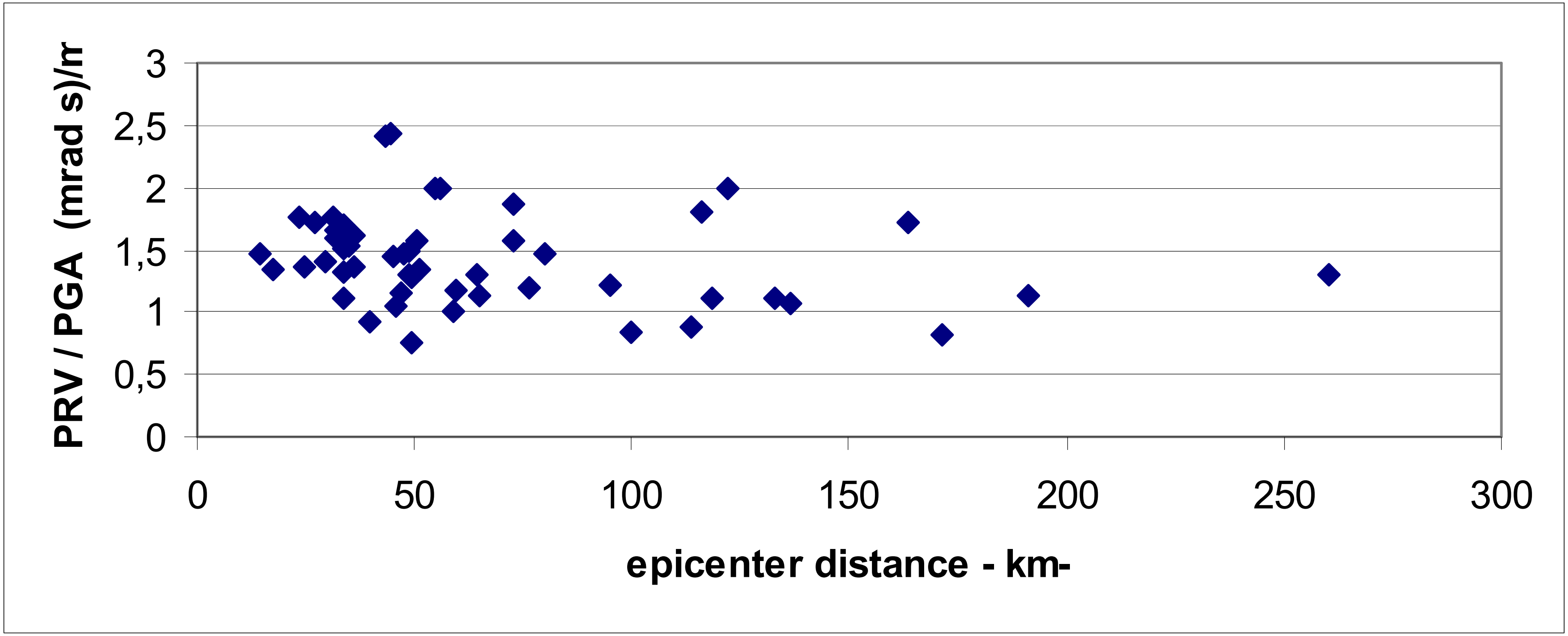
Direct measure

- Rotational ground motions have not been observed directly until the last decade
- **Rotation-sensitive instruments** have been developed to provide the evaluation of the rotation at a point

Indirect measure

- Rotations obtained through records collected by **closely spaced arrays** of strong motion accelerometers
- Spatial distribution of the translation described through the **Cross Power Spectrum of translations**
- **Mathematical relationship** between the Cross Power Spectrum and the Power Spectrum of rotations

Direct measure

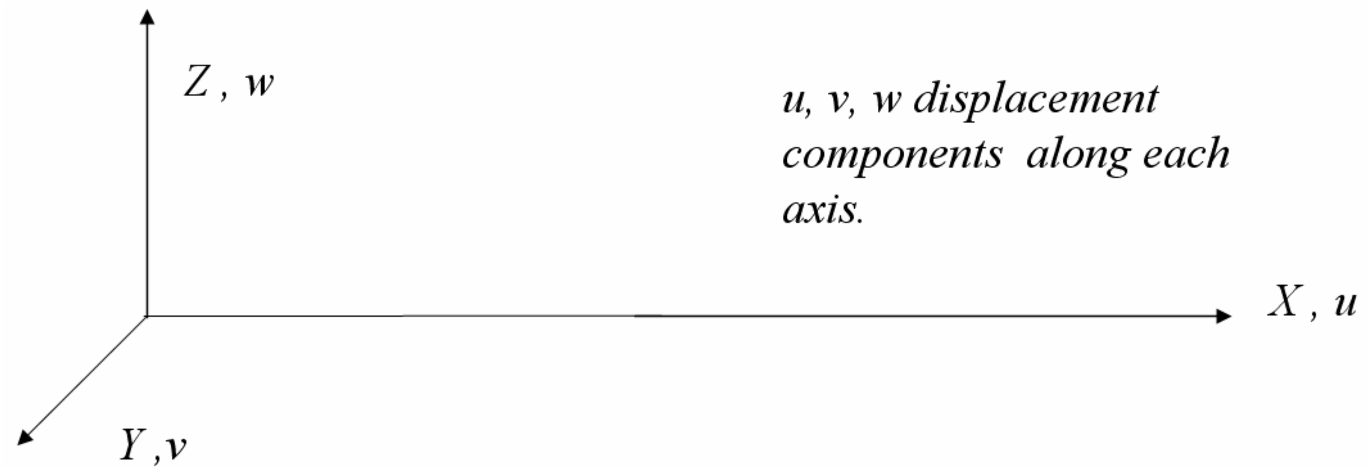


Data presented by Liu et al. 2009. Ratio between the peak rotation velocity PRV, and the peak horizontal acceleration PGA, in mrad × s/m. Average value 1.43.

Power Spectrum of Rotations

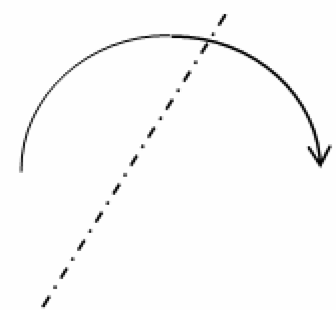
Mathematical relationship between the rotational power spectrum and the cross power spectrum simultaneously registered in an array

Definitions



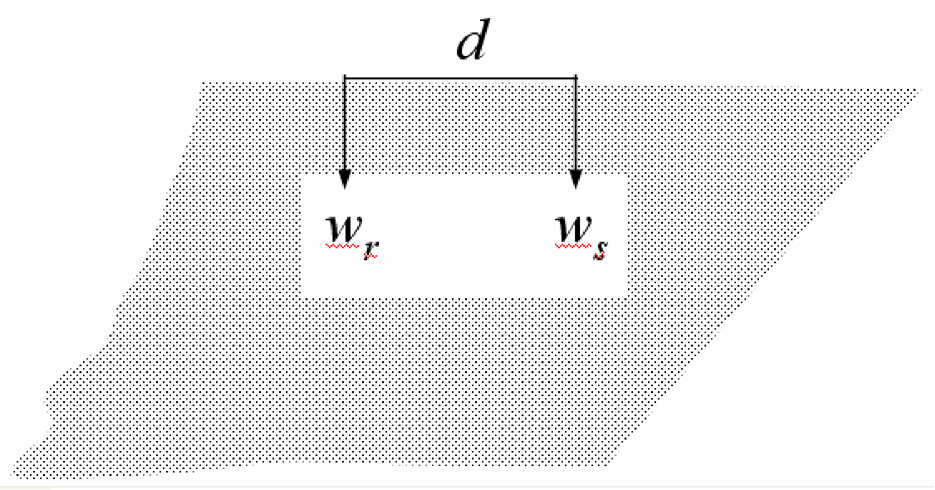
- Rotation around the y-axis

$$\psi_y = \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)$$



- Cross-Power Spectrum of 2 vertical signals w_r, w_s at distance d

$$S_w(\omega,d)=\lim_{T\to\infty}E\left(\frac{2}{T}\left[W_r(\omega)\times W_s^*(\omega)\right]\right)$$



$$\begin{cases} W_r(\omega)=\int_0^T w_r(t)e^{-i\omega t}dt \\ W_s(\omega)=\int_0^T w_s(t)e^{-i\omega t}dt \end{cases}$$

- Coherency function
[Abrahamson et al, 1985]

$$\gamma_w(\omega,d)=\frac{S_w(\omega,d)}{S_w(\omega,0)}$$

- Average rotation between 2 points r and s separated by distance d

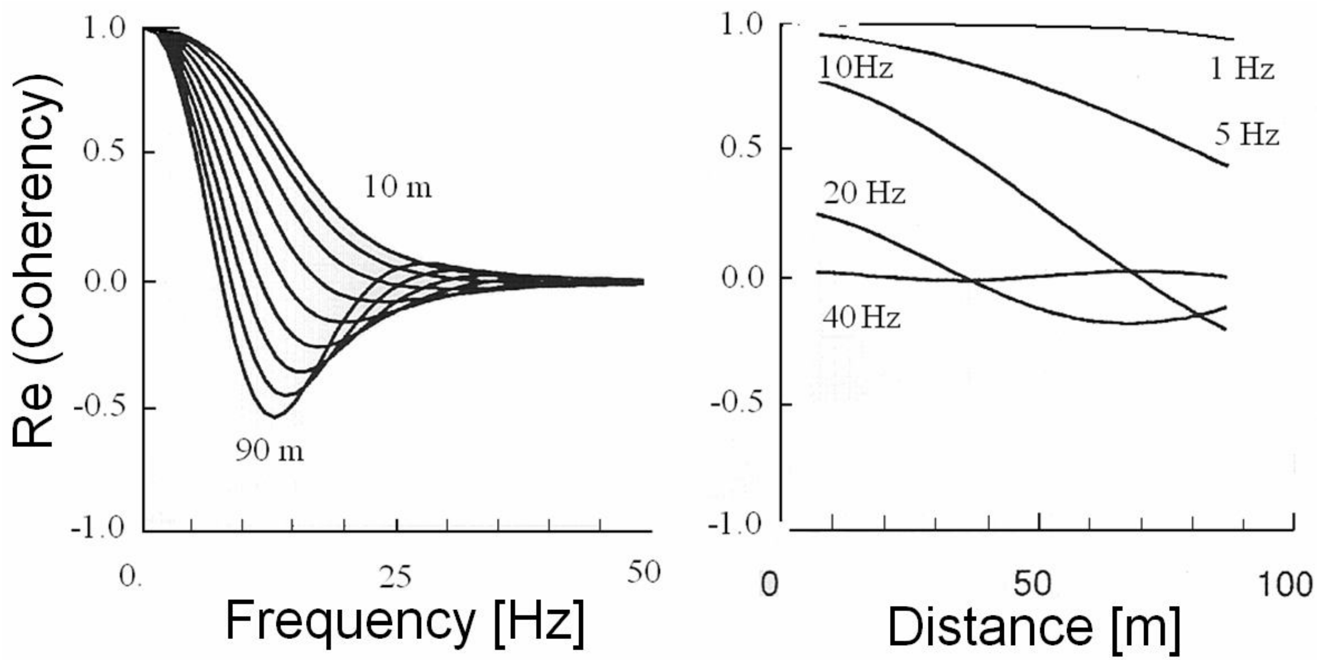
$$\psi_d(t)=\frac{w_r(t)-w_s(t)}{d}$$

- Power Spectrum of rotation

$$S_\psi(\omega,d)=\frac{2S_w(\omega,0)}{d^2}\left(1-\mathrm{Re}\left[\gamma_w(\omega,d)\right]\right)$$

- Power Spectrum of rotation at a point

$$S_\psi(\omega,0)=\lim_{d\to 0}S_\psi(\omega,d)$$



Real part of coherency, function of frequency and inter-station separation.
Records of accelerations.
[Abrahamson et al, 1991]

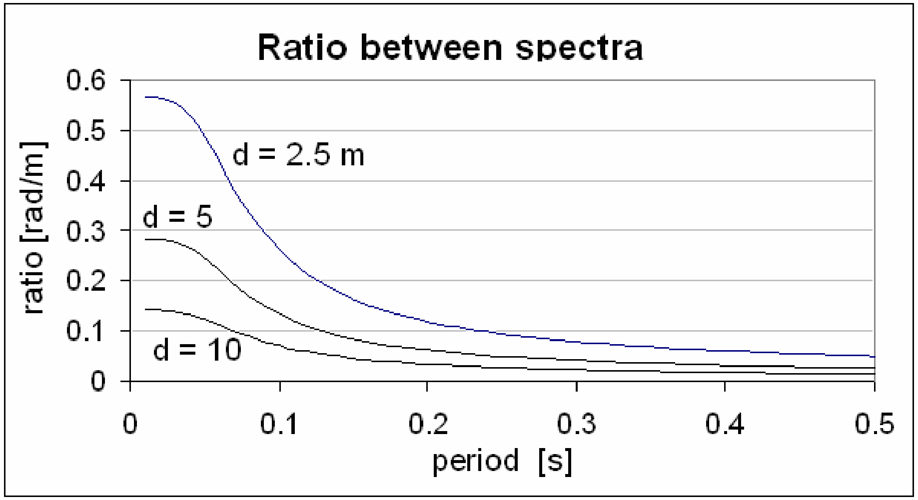
Dependence of the measurements on the separation d

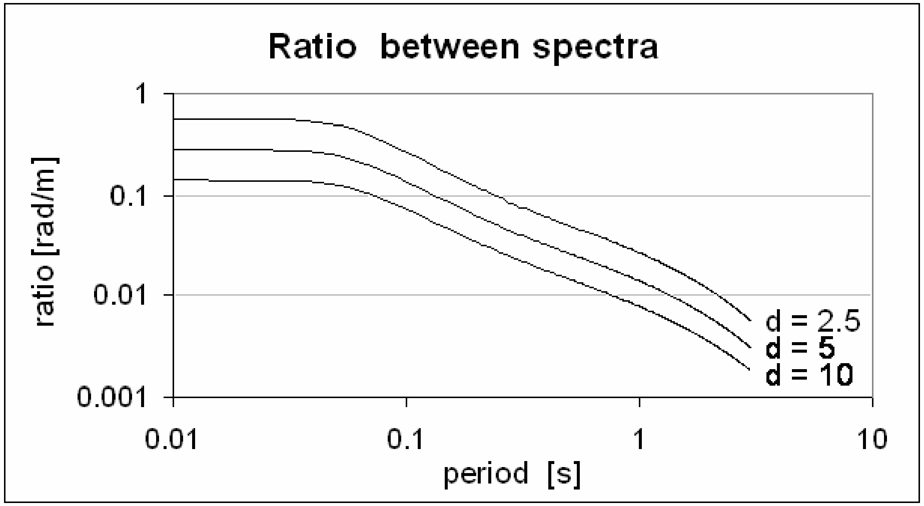
- The ratio between power spectra tends to 0 for increasing periods

$$\sqrt{\frac{S_{\psi}(\omega,d)}{S_{w}(\omega,0)}}$$

- The cross power spectrum between closely-spaced stations depends on separation between stations d

- Rotation of mass-less rigid foundation assumed to be related to a rotation averaged over a separation comparable to the block dimensions





Rotational accelerogram from measured vertical accelerogram

- Ground rotational acceleration records few and affected by noise with uncertainties

- Evaluation of rotational accelerogram on the basis of the relationship between rotational and vertical spectra

Input vertical ground acceleration written as Fourier series

$$\ddot{w} = \sum_{1}^{N} a_i \cos\left(\omega_i t + \varphi_i\right) = a_1 \cos\left(\omega_1 t + \varphi_1\right) + a_2 \cos\left(\omega_2 t + \varphi_2\right) + \ldots$$

$$\ldots + a_i \cos\left(\omega_i t + \varphi_i\right) + \ldots + a_N \cos\left(\omega_N t + \varphi_N\right).$$

$$\omega_1 = \frac{2\pi}{T} \text{ rad/s}, \ \omega_2 = 2 \times \omega_1, \ \ldots, \ \omega_i = i \times \omega_1, \ \ldots, \ \omega_N = N \times \omega_1$$

- Input **vertical power spectrum**, defined at discrete points ω_i

$$S_w = \left(\frac{a^2}{d\omega}\right) = a^2 \frac{T}{2\pi}$$

Output rotational acceleration written as Fourier series

- Relationship between rotational and translational power spectrum, defined at discrete points ω_i. Ulagged-coherence method by Abrahamson

$$S_\psi = \frac{2S_w}{d^2}\left(1 - cohe\left(\omega, d\right)\right)$$

- Rotational accelerogram

$$\ddot{\psi} = \sum_{0}^{N} h_i \cos\left(\omega_i t + \varphi'_i\right)$$

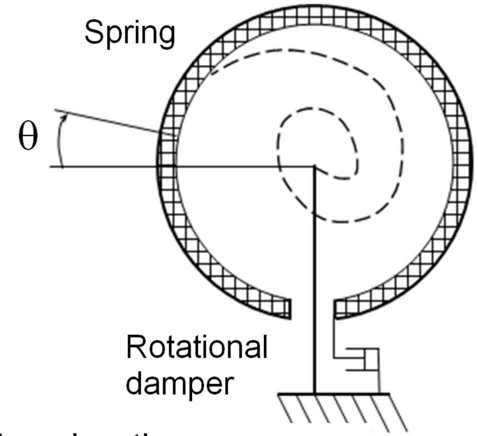
with:

$$h_i = \sqrt{S_\psi \cdot d\omega} = \sqrt{S_\psi \cdot \frac{2\pi}{T}}$$

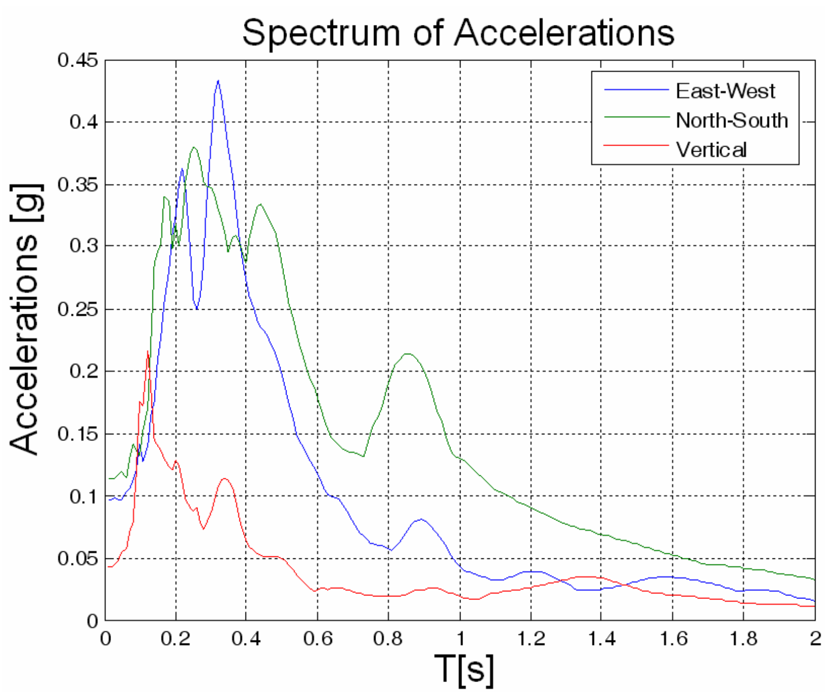
Phase factors

- Same factors of the vertical component
- Peak values in correspondence with input vertical ground acceleration peaks

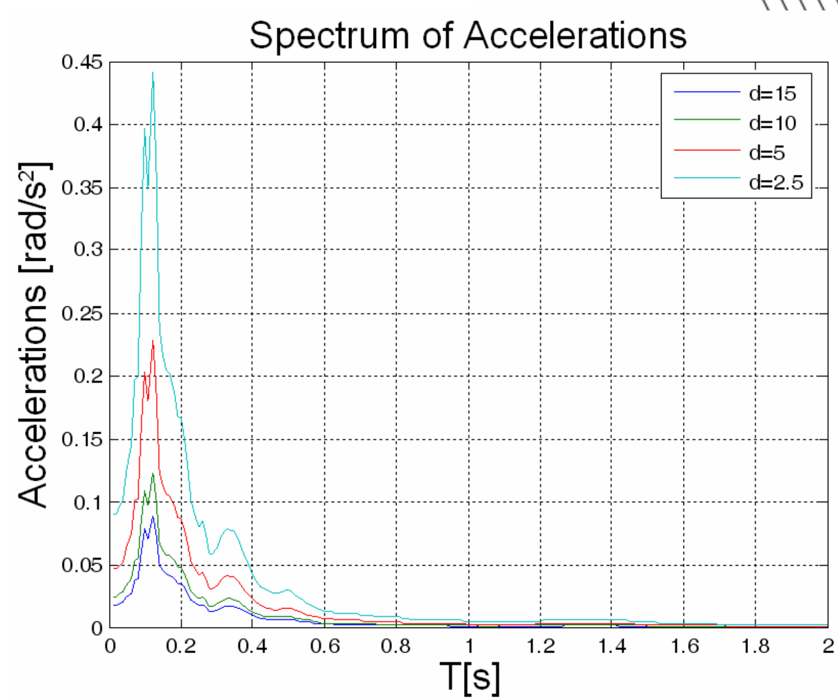
Definition of Rotational Response Spectra



- For a one degree of freedom oscillator, it represents the peak rotational acceleration of the oscillator, during the earthquake duration



Translational Response Spectra
Taiwan SMART1(5)C00 29/01/1981



Rotational Response Spectra,
varying the value of separation d

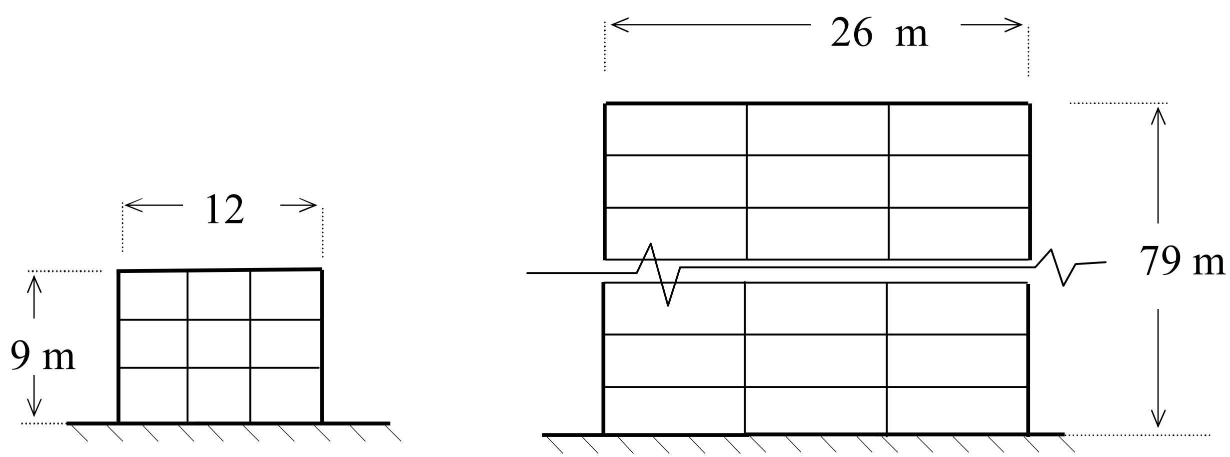
Aim

• Illustrate the relevance of rotational input motions with respect to the effects of horizontal excitations consistent with them

Assumptions

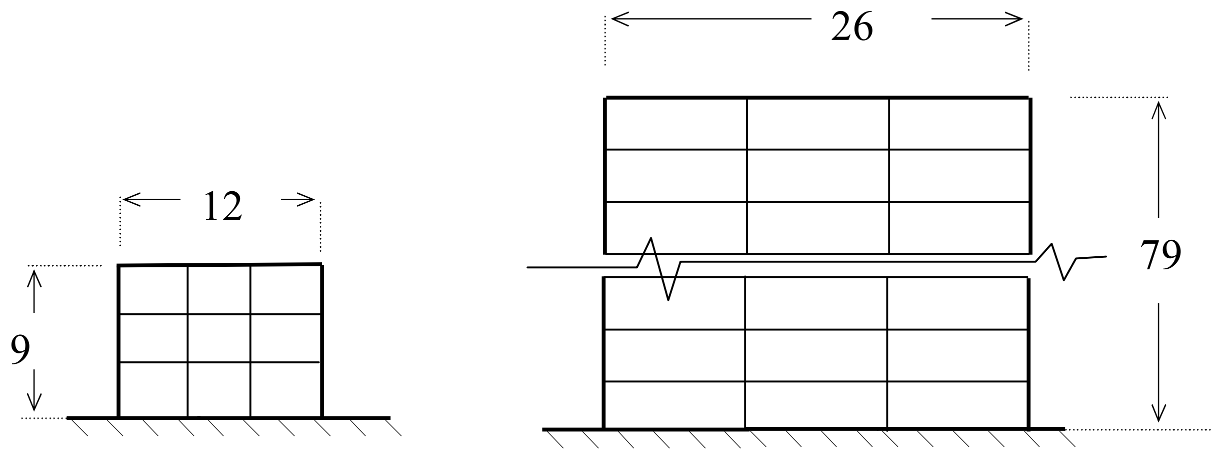
• Free field rotation applied to the foundation without modification from reaction forces, disregarding soil compliance

• Rotation estimate according to a separation d, suitable to represent the global effects on the building



Model

- Identical mathematical model applied in the analysis of the translational and rotational excitation
- Two simple 2D model buildings, of a moment-resisting structural type
  - Reinforced concrete three stories structure *(First natural period T=0.48 s)*
  - Steel twenty stories structure *(First natural period T=1.56 s)*



Results

• Evaluation of the **relative incidence** (in percentage) of the rotational component to the translational one, on the basis of the roof horizontal displacement

• Several analysis considering different **real earthquakes** (*referring to Taiwan SMART1(5) and Taiwan SMART1(33)*)

		Rotational Component Incidence [%]			
		East-West		North-South	
	Frame	*Max*	*Min*	*Max*	*Min*
Taiwan SMART1(5): C00	**3 x 3**	*1.37*	*1.52*	*0.96*	*1.27*
	20 x 3	*21.63*	*17.52*	*9.39*	*9.09*
Taiwan SMART1(5): O01	**3 x 3**	*1.50*	*1.67*	*0.61*	*0.65*
	20 x 3	*11.65*	*12.2*	*5.54*	*6.07*
Taiwan SMART1(33): C00	**3 x 3**	*1.57*	*1.91*	*1.60*	*1.78*
	20 x 3	*14.30*	*18.51*	*12.07*	*13.83*
Taiwan SMART1(33): M07	**3 x 3**	*0.81*	*0.66*	*1.45*	*1.27*
	20 x 3	*8.90*	*10.08*	*9.08*	*10.83*
Taiwan SMART1(33): I01	**3 x 3**	*1.56*	*1.77*	*1.37*	*1.51*
	20 x 3	*12.29*	*13.25*	*9.96*	*10.84*

Results

- For stiffer building, of the same height, the contribution of rotation will be greater
- For **squat building** the rotation effects fluctuate within the range of uncertainties connected to earthquake motions, and thus are of **little concern**
- The contribution of **vibration modes** higher than the first is minor
- **Higher ratio** between rotational and translational effects for **tall building** model, in some case incidence higher than 20%
- Key assumption is that soil deforms as in **free field**, without soil-structure interaction
- For Taiwan records the ratio between vertical and horizontal peaks is 03 - 0.5 while in other near field situation a ratio even **higher than 1** has been observed,

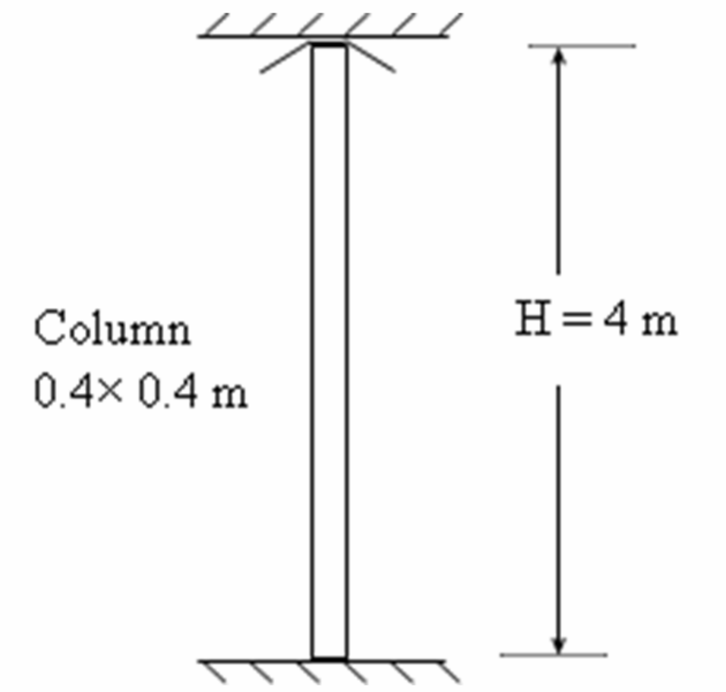
Local effects

• Only one element assumed for columns between two stories and beams within a single bay

• Local effects, in which columns or beams vibrate along the spam, thus disregarded

• Possible vibrations of columns can be investigates with simple structural model that highlight the effect of high frequency part of rotational response spectrum

• To relevant peak accelerations, correspond small peak displacement and thus tiny resultant stress



➢ From a structural point of view, if a vertical motion different from point to point is superposed to a rigid translation of the soil, there are **two possible consequences**

1. *High frequency vibration along beams and columns*
2. *Overall rotation of the base of the structure*

➢ First effect capable to provide huge local accelerations, but meaningless for resistance of the structure

➢ **Stress** due to the second effect tends to increase with the building height and with the stiffness of the building

➢ If the plane dimensions are comparable to the building height, rotations effects can be disregarded

➢ For relatively tall buildings having ratios of height to plan dimensions around 3 or more rotations in near field should not be disregarded

Thank you for your attention