Inverting Ground Motion from a Seismometer Array to Obtain the Vertical Component of Rotation: A Test Using Data from Explosions

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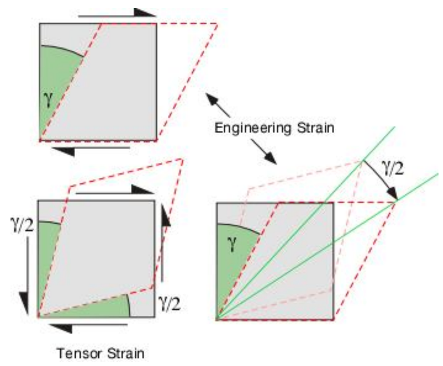
Outline

- A new inversion method based on Jeager's formula
- Spudich and Fletcher's method
- TAIGER experiment (Explosion: 10 tran. + 5 rot.)
- Inverting Explosion Data from TAIGER experiment (compared with the R1 waveforms)
- Synthetic waveform tests with different levels of noises
- Conclusions

Jaeger (1969): Forward Problem



$u = e_{xx}\, x + (e_{xy} - w)\, y$

$v = (e_{xy} + w)\, x + e_{yy}\, y$

- where e_{xx} and e_{yy} are the normal strains in the x- and y-directions, respectively.
- The e_{xy} is the shear strain, and w is the rotation.
- There are no constraints on these equations imposed by the free surface.
- Because we have observations of u and v at numerous stations, we can simultaneously invert those equations for the time derivatives of e_{xx}, e_{xy}, e_{yy}, and w.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned}$$

Inverse Procedure:

Based on Formula of Johnson and Wichern (1988) to Estimate Errors

$$\mathbf{Y} = \mathbf{Z}\,\beta + \varepsilon$$

Where

ε is the error

E(ε)=0

β is the unkown: e_{xx}, (e_{xy}-ω), (e_{xy}+ ω), and e_{yy}

Z is related to spacing between stations

Y are related to recorded differential translational ground motions

Invert to minimize (**Y** - **Z β**)' * (**Y** - **Zβ**)

$$\mathbf{Y} = \mathbf{Z}\beta + \boldsymbol{\varepsilon}$$

Y is an n×2 matrix composed of measurements u and v (from seismometers).

The u and v at each particular time step have a covariance matrix,$\{\sigma_{ik}\}$, i.e., the covariance

$\mathrm{Cov}(\boldsymbol{\varepsilon}_{(i),}\ \boldsymbol{\varepsilon}_{(k)}) = \sigma_{ik}\,\mathbf{I}$

where i, k = u or v, and **I** is the identity matrix

To Estimate Errors

First derive the variance of ω

$$
\begin{aligned}
&\mathrm{Var}(\omega) \\
&= \mathrm{Var}(\ ((e_{xy} + \omega) - (e_{xy} - \omega)) / 2\ ) \\
&= 1/4 * (\ \mathrm{Var}(e_{xy} + \omega) \\
&\qquad + \mathrm{Var}(e_{xy} - \omega) \\
&\qquad - 2 * \mathrm{Cov}((e_{xy} + \omega), (e_{xy} - \omega)))
\end{aligned}
$$

obtained from Result 7.9 of Johnson and Wichern (1988) in terms of **Z** and σ_{ik}.

Standard Deviations = $(\mathrm{Var})^{0.5}$

Spudich and Fletcher (2009)

$$d^{i} = u^{i} - u^{0} = GR^{i}$$

Known (measured):

d^i: displacement differences between translational stations

R: pre-disturbance offset of stations

Unkown:

G is a 3X3 displacement gradient matrix containing the unknown strain and rotation information we want to derive

Matrix Manipulation (Spudich and Fletcher, 2009)

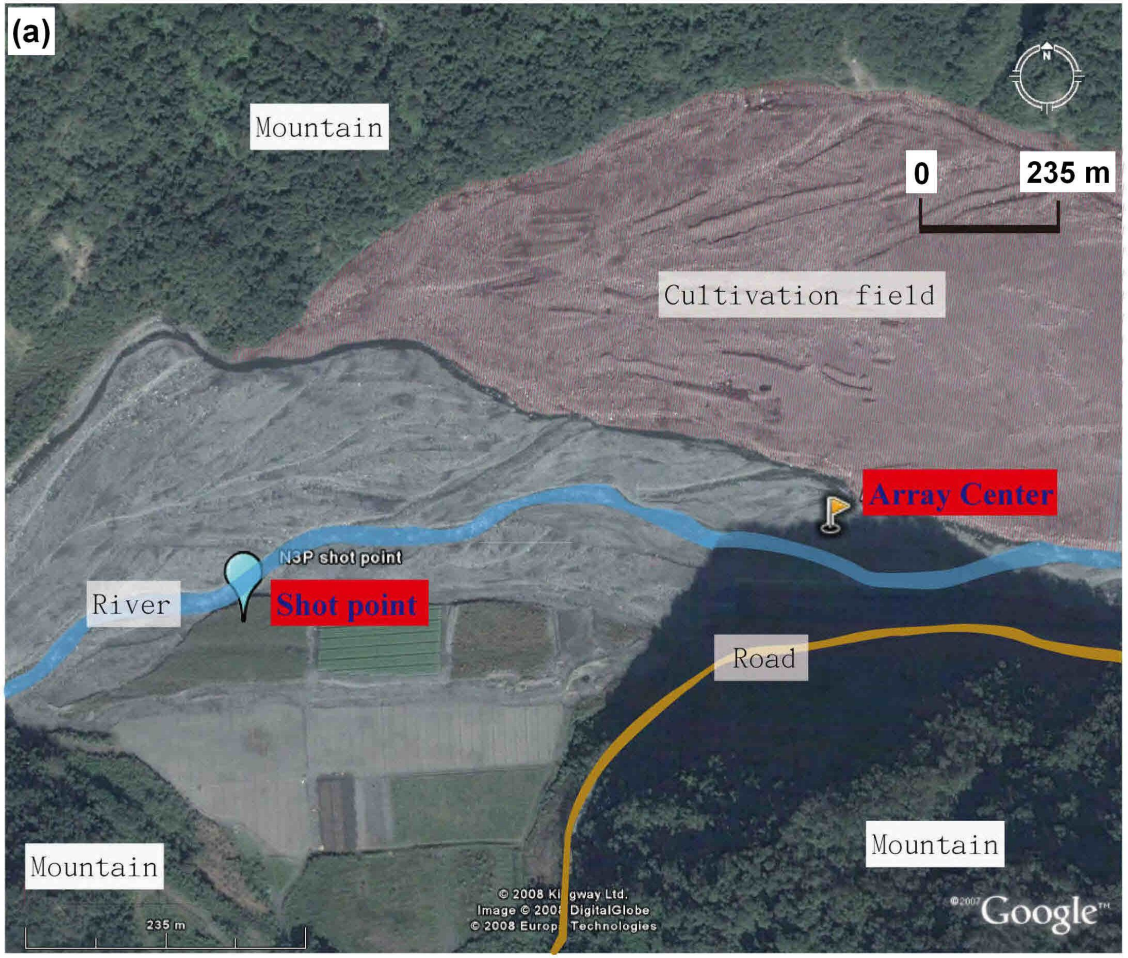
$$d^i = u^i - u^0 = GR^i$$

- Because the Z-component of the stress tensor is zero at the free surface, The G tensor can be reduced to a 5-element vector *p*.

$$d^i = A^i p^i$$

A matrix is related to the matrix R (relative station locations) and the Lamé parameters of the materials underneath the array.

Now we can use a classic inversion method to invert for **d**.



Now the Field Data:

TAIGER Experiment

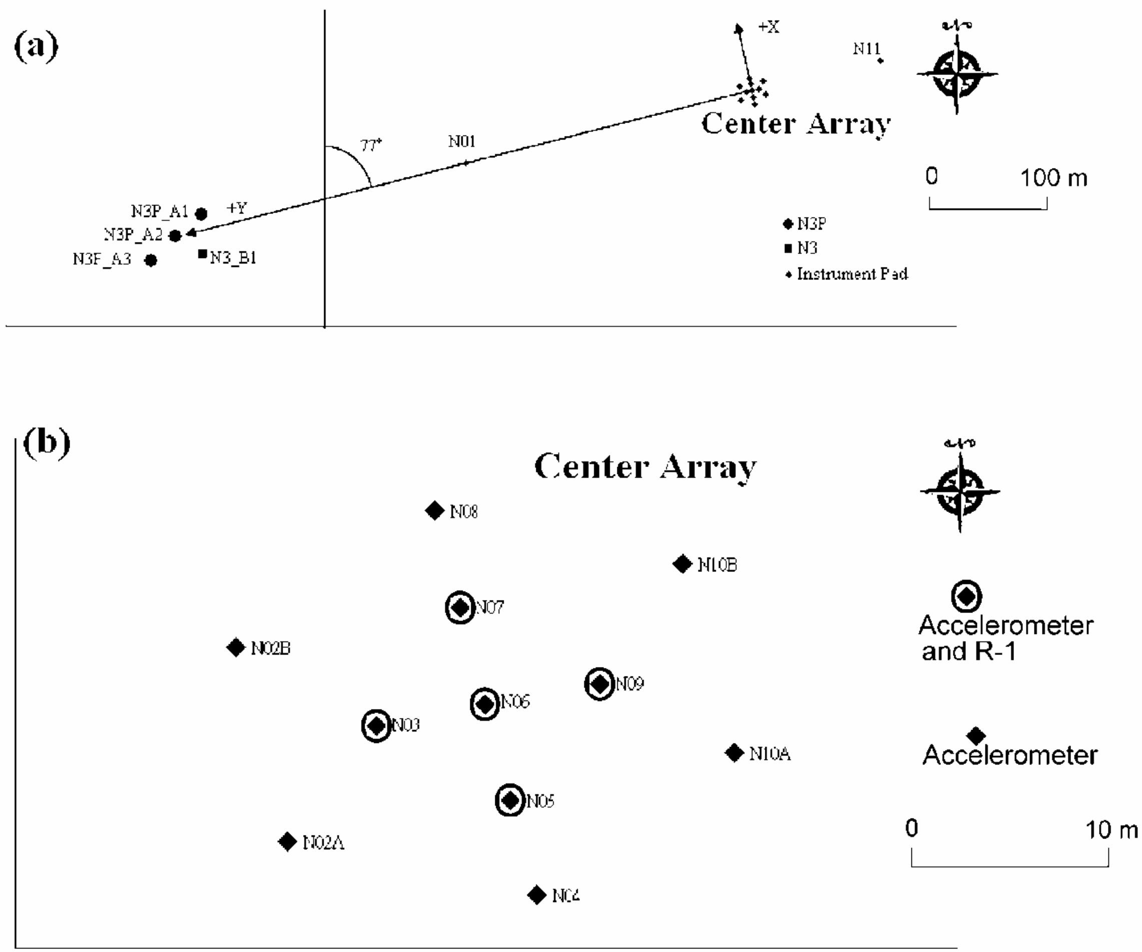
Explosive: 1500 kg

Shot to array: 500 m
Station spacing: 5 m

Accelerometers: 10
R1 rotational sensors: 5

Lin et al. 2009, BSSA

Station Configurations

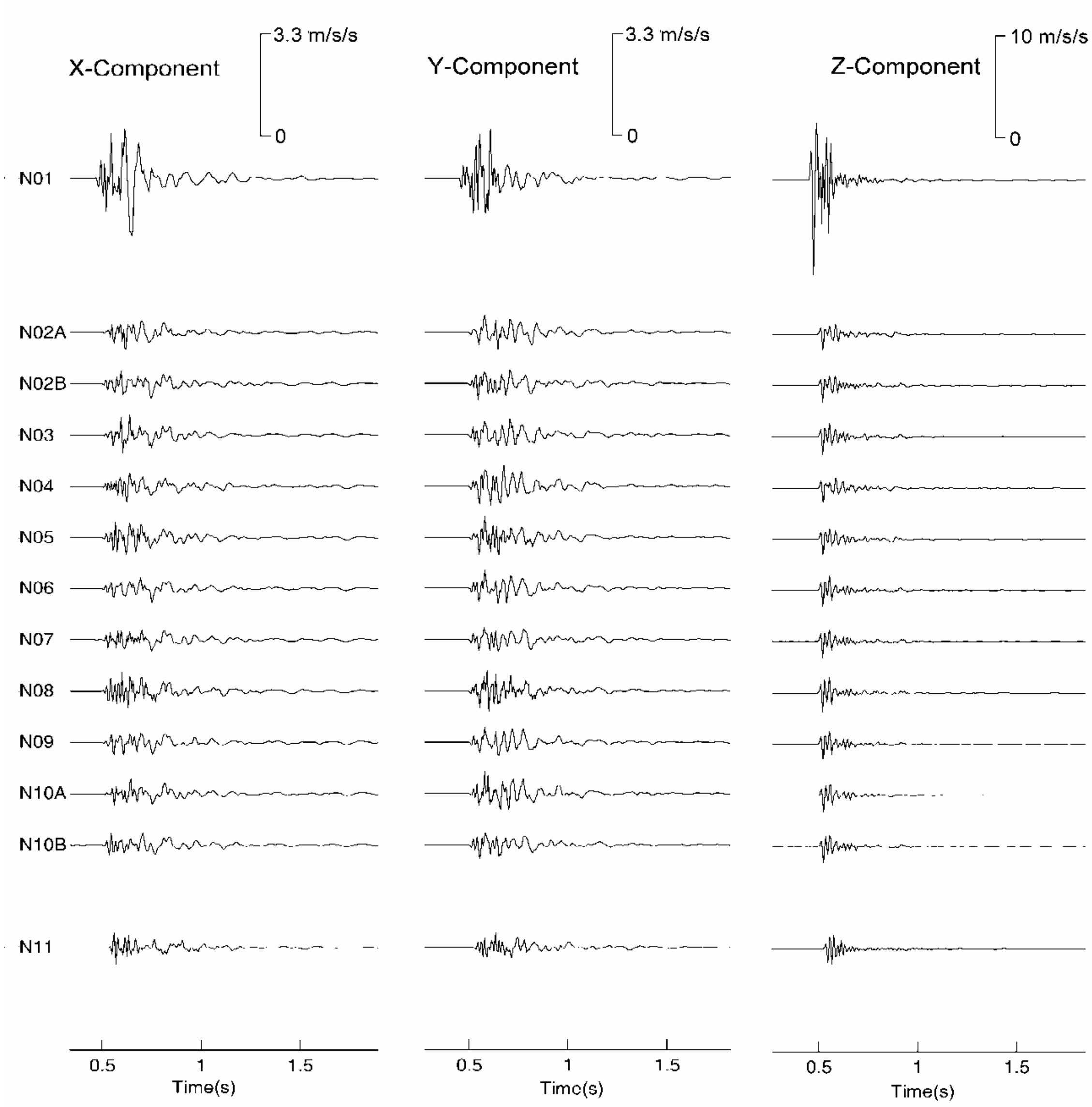


Lin et al. 2009, BSSA

Martin's poster yesterday shows many interesting results:
Invert for 3 rot. components
FK analyses
Dispersion Study

Translational Acceleration Waveforms (N3 shot)

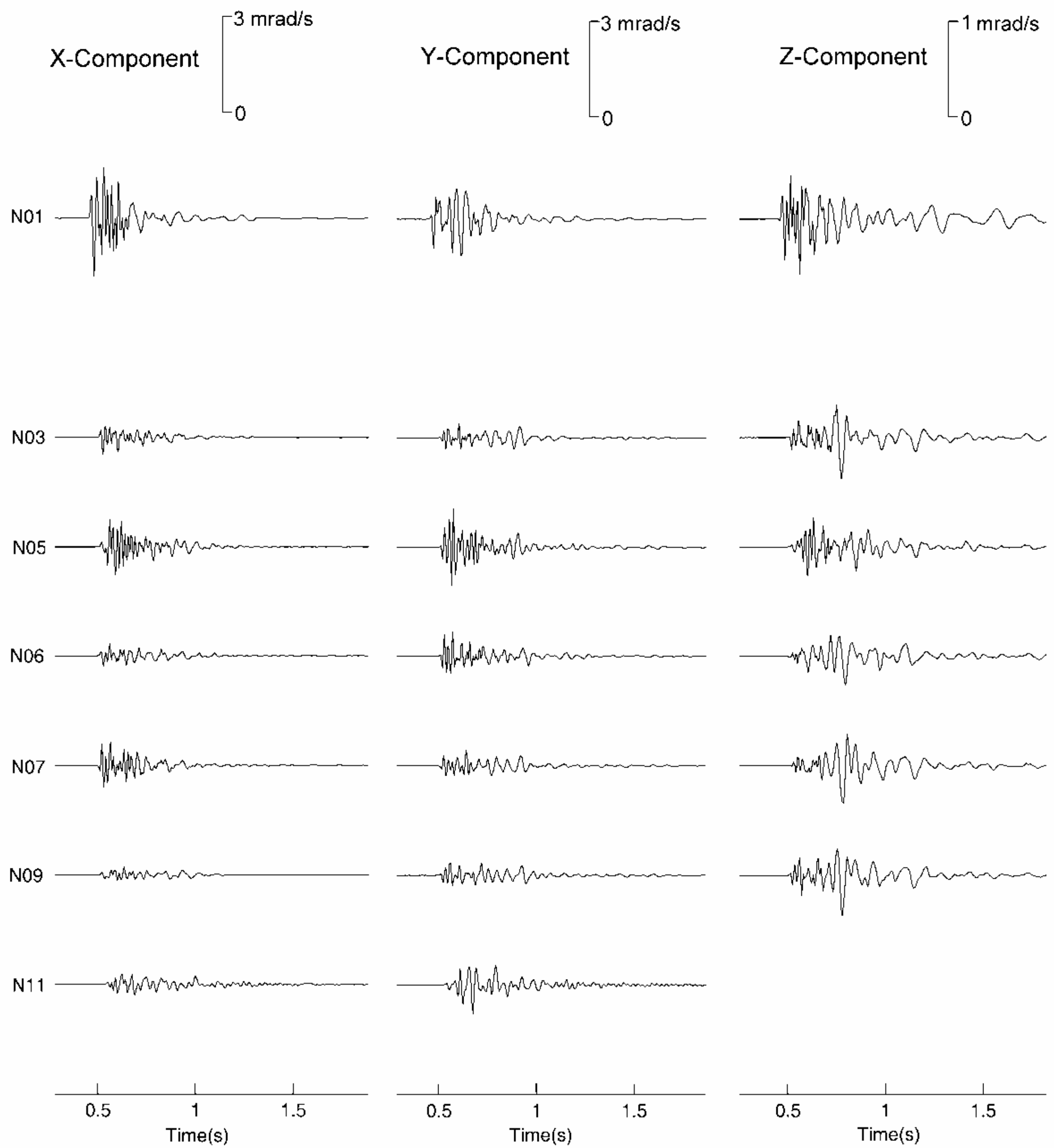
250 m



500 m

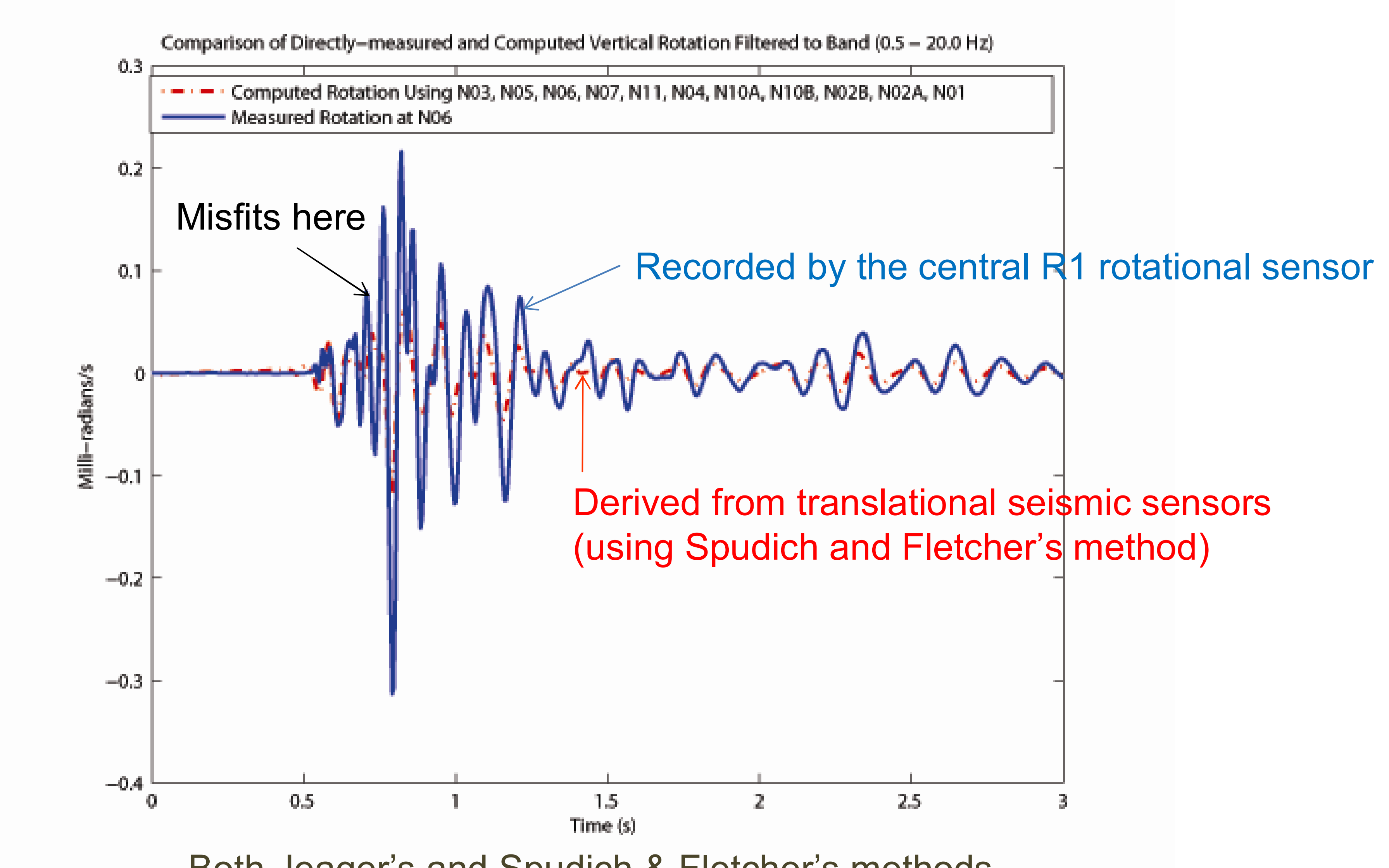
Consistent waveforms

Rotational Rate Waveforms



Z components are more consistent

Inverted Results: Rotation Waveform Fits (0.5-20 Hz)



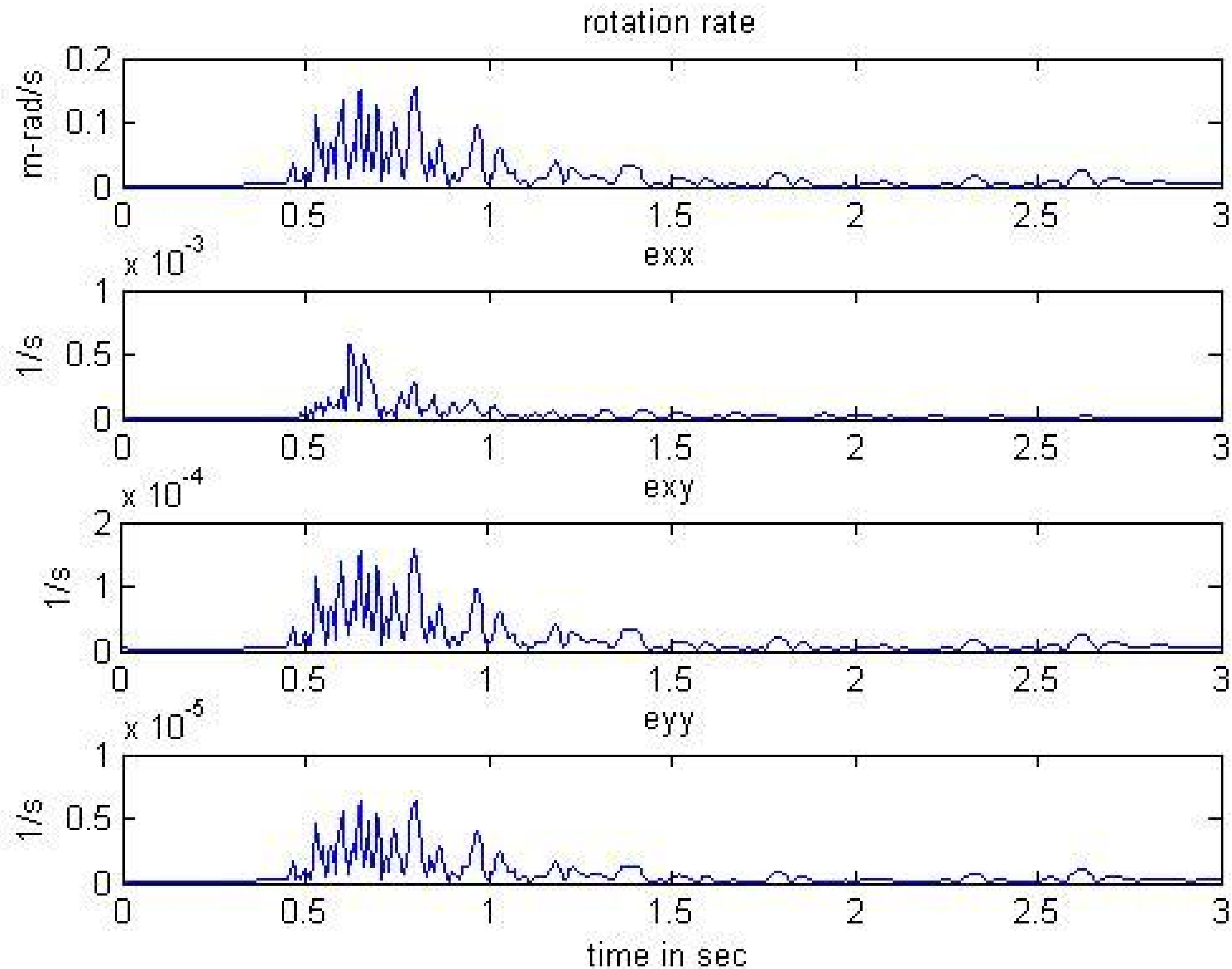
Both Jeager's and Spudich & Fletcher's methods derived similar results

Why misfits in the beginning?

Large strains?

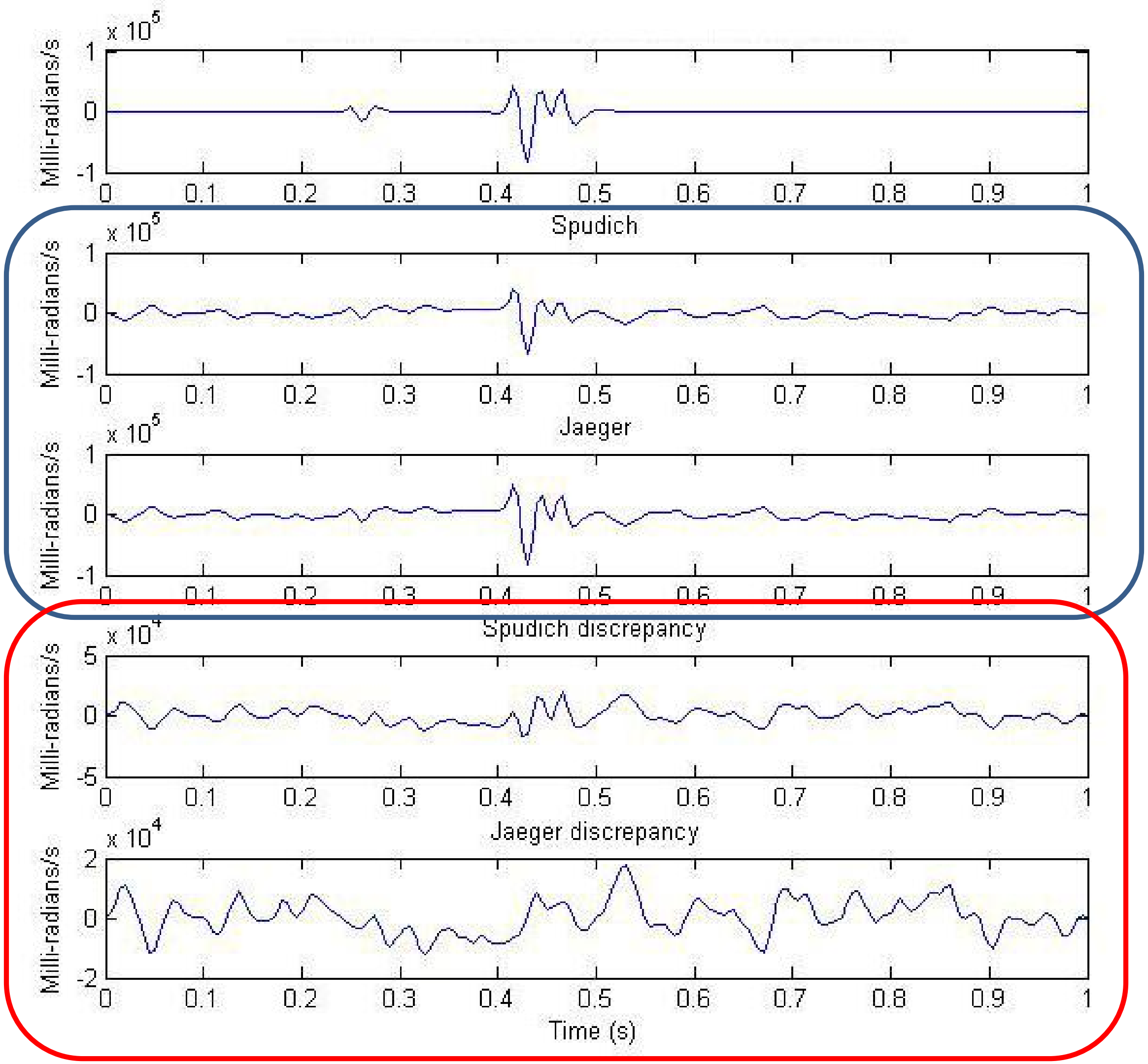
Others: heterogeneous medium, noises?

Large Strain Rates Excited by the Explosion



Next we use synthetic translational waveforms to study the performance of the inversion codes on handling different noise levels of the translational waveforms

Synthetic Translational Waveform Tests

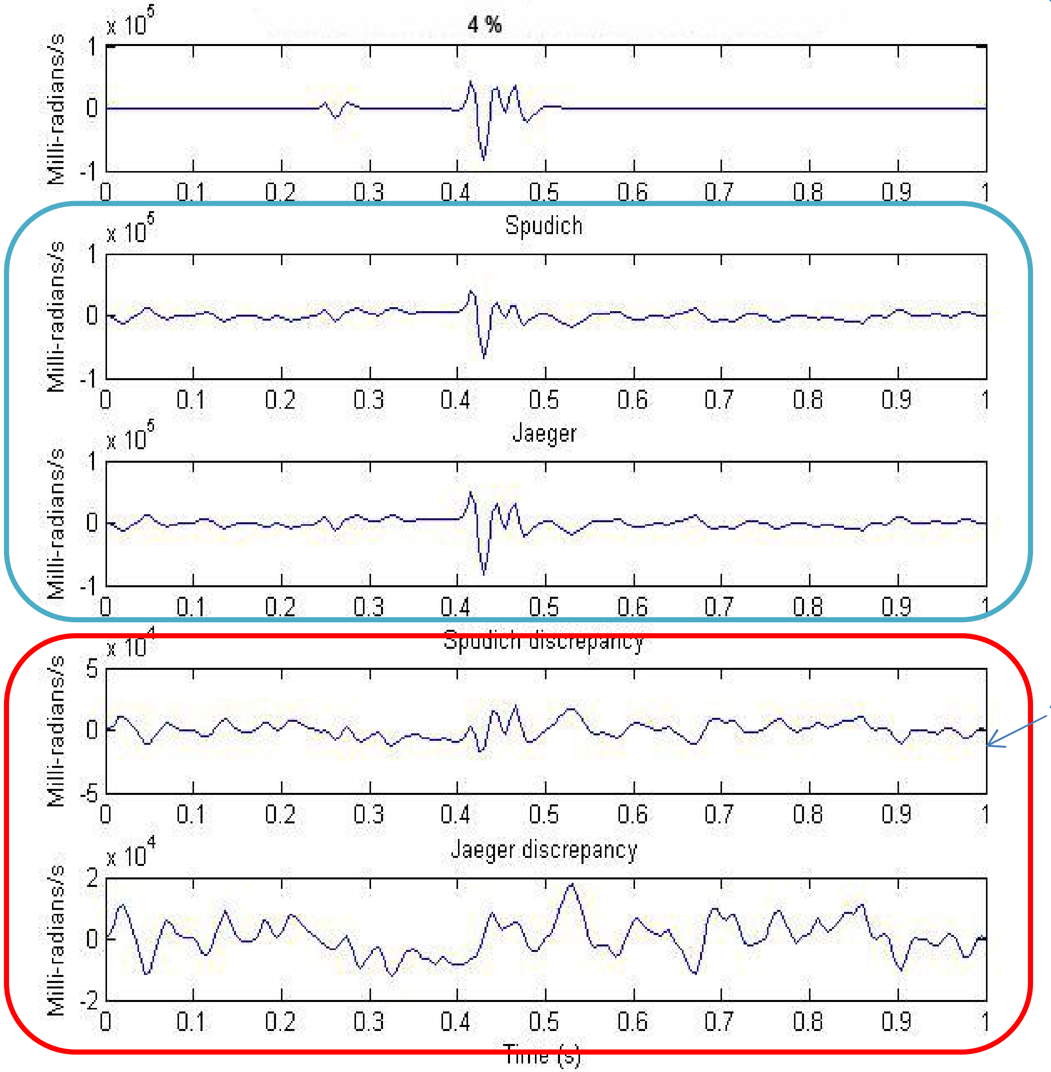


3 component translational waveforms are generated using FK method by Zhu (2003)

An explosion source in a half space crustal velocity model

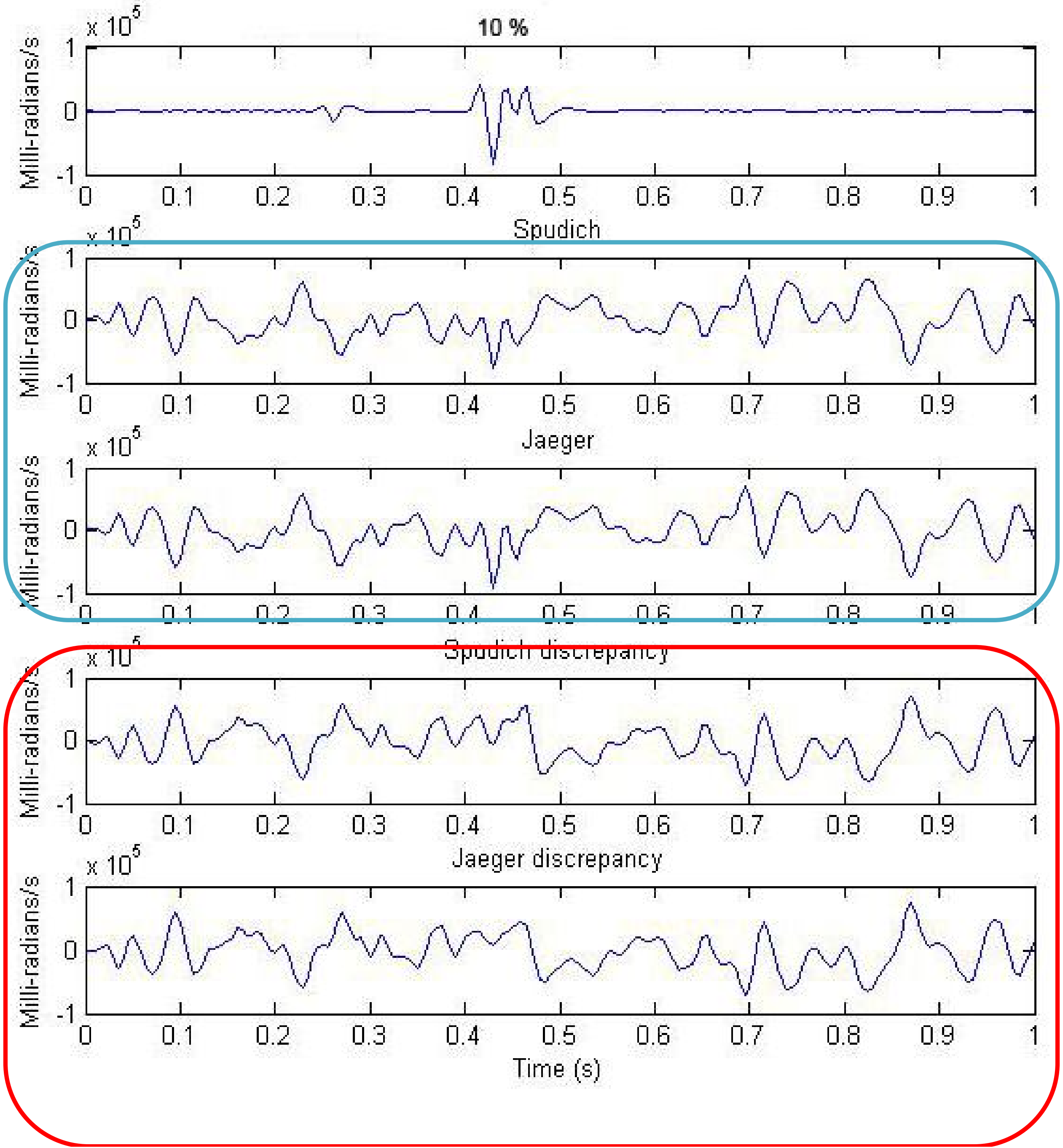
Again, both methods derived very similar results

The inversions are sensitive to the noises in the translational waveforms (consistent with previous studies from Prof. Igel's group)



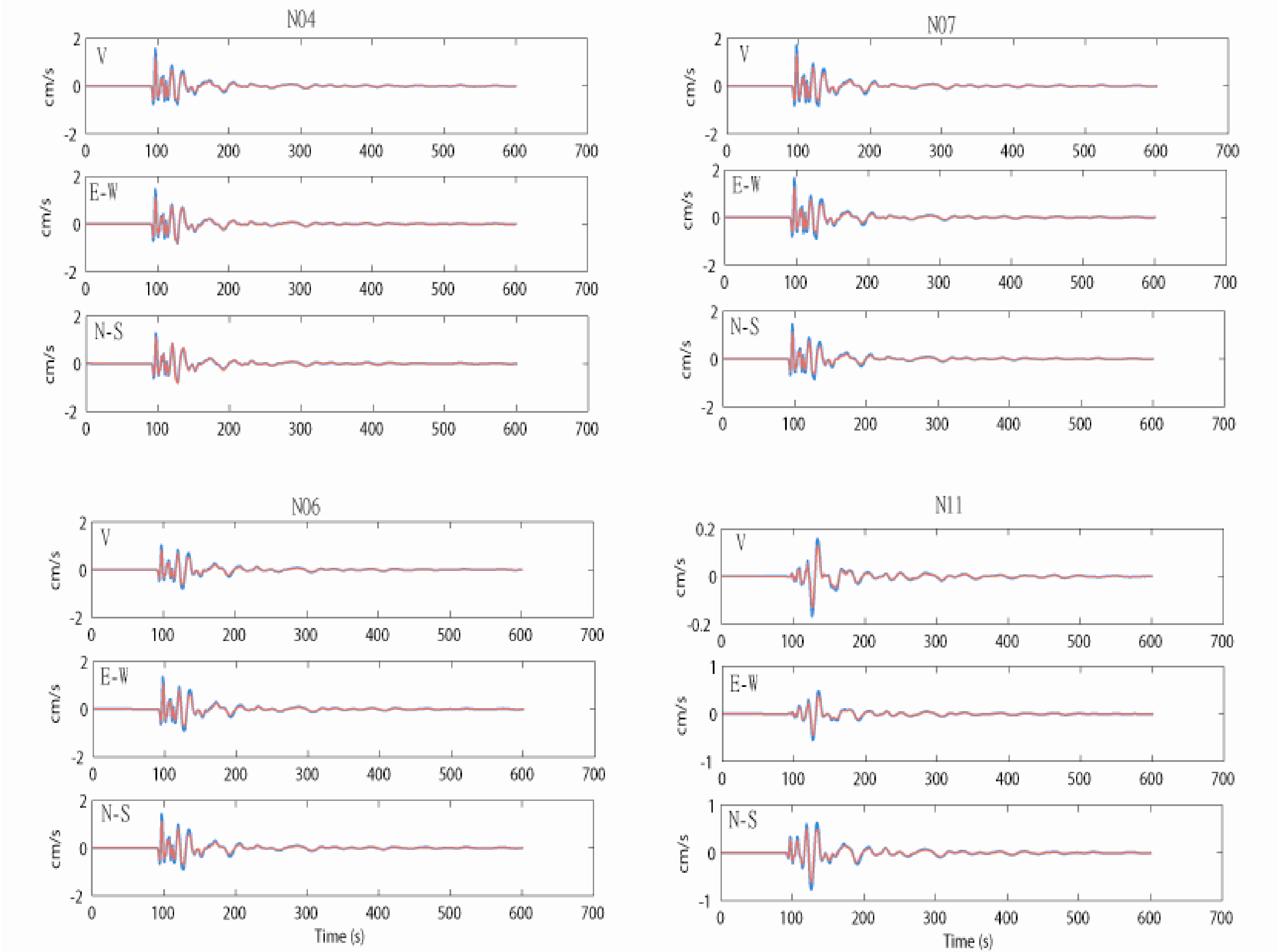
4%: based on pre-event noise of the TAIGER translational data

A bit greater discrepancy using Spudich and Fletcher's method (for this case)



Inversions become difficult after adding 10% of noises to the translational synthetics (for both methods)

Using our rotation waveforms (inverted results) to predict Z translation components (excellent predictions, why?)



Comparison Between the Two Methods

Jeager's method:

- Can only work on 2D problems one at a time (can do it three times consecutively to do a 3D problem)
- No need for free surface constraints
- Do not need Lame constants a priori
- Can estimate the errors of the waveforms using different statistic methods

Spudich and Fletcher's method:

solves 3D problems in one shot

well tested.

Summary

1. a new way to invert translational waveforms for rotational ground motions (Jeager, 1969) and (Johnson and Wichern, 1988).
2. Results are consistent (a bit better) with the results from Spudich and Fletcher's code, possibly due to less a prior assumptions required by the new code.
3. Both codes gave consistent results for TAIGER explosion dataset (Especially 0.5 to 5 Hz band).
4. Using synthetic translational datasets we found that we can use array data to invert for rotational ground motions, if the translational waveforms are not too noisy.

(submitted to BSSA, under revision)