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Seismic Earth Rotations Observed with Inertial Seismic Sensors

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Inertial sensors along one direction

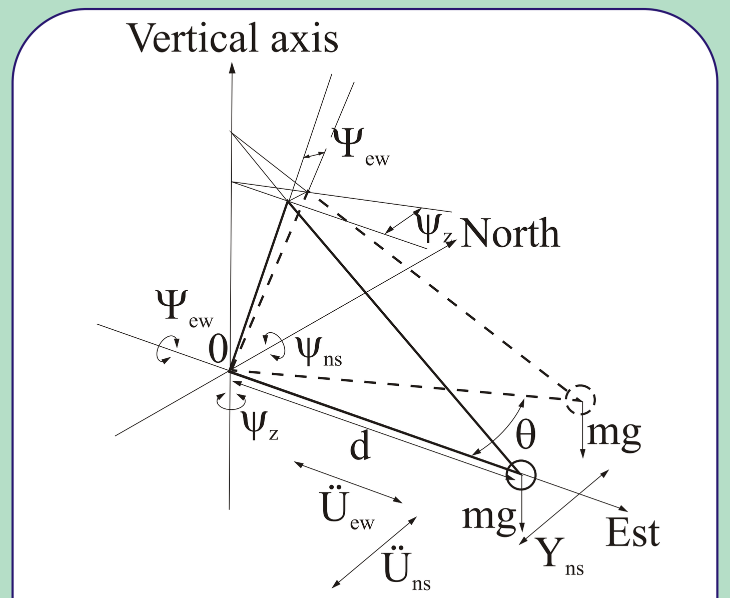

Fig 1 : Schematic description of the garden-gate seismograph with four degrees of freedom

'Garden-Gate' Horizontal Pendulum

The `garden-gate' horizontal pendulum has four possible degrees of freedom for the mass motion: this is the simplest seismograph which takes into account rotation effects. Let us consider a North-South horizontal component which means that the mass is along the East-West direction.

- The first degree of freedom is the horizontal translatory motion U_{ns} of the Earth.

- The second degree of freedom is the rotation Ψ_{ns}. This rotation and the associated angle controls the period of the pendulum.

- The third degree of freedom is the rotation Ψ_{ew} often named as tilt. This tilt and the associated angle controls the zero position of the mass.

- The fourth degree of freedom of this type of pendulum is a rotation around the vertical axis, denoted Ψ_z.

The angle Θ (or the Y_{ns} translation) depends within the first order on three degrees of freedom: two rotations and one translation. Considering works of Rodgers (1968), Bradner and Reichle (1973) and Graizer (2005, 2006), the equation defining the measurable parameter Y_{ns} can be written as

$$\ddot{Y}_{ns} + 2\beta\omega_0\dot{Y}_{ns} + \omega_0^2 Y_{ns} = -\ddot{U}_{ns} + g\Psi_{ew} - d\ddot{\Psi}_z \qquad (1)$$

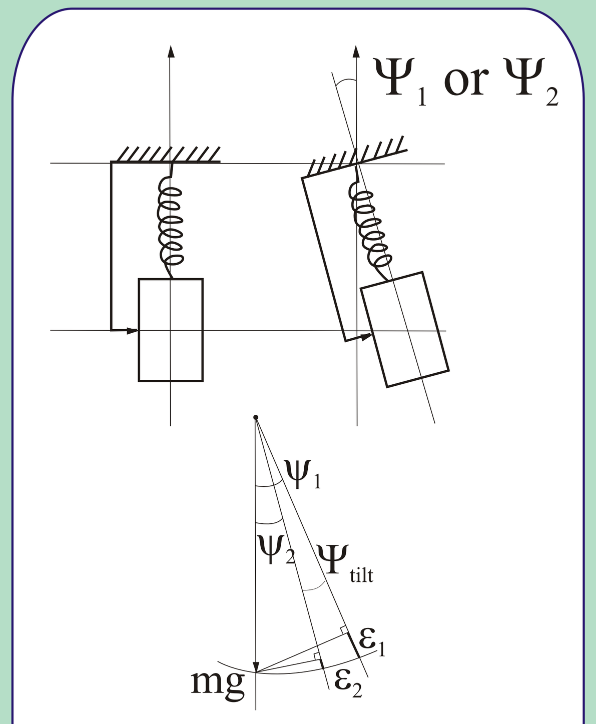

Fig 2 : Tilted vertical seismograph. Combining tilt effects should take into account the non-linearity of these effects

Vertical Seismometer, Geophone Type

We consider a vertical geophone built with a cylindrical mass guided in its translation by six elastic fasteners (piano wires) as for seismometers of Benioff and Willmore.

The geophone may be correctly installed with a vertical axis along the translation axis as shown in figure 2. The installation could be such that the translation direction is tilted by the angle Ψ_1 from the vertical. This geophone may suffer additional tilt defined by the angle Ψ_2.

When considering a tilted sensor, the moving mass undergoes a force deficit denoted ε_1 as the weight vector (m.g) is projected on the new sensitivity axis giving us the following equation:

$$\frac{mg - \varepsilon_1}{mg} = \cos(\Psi_1) \Rightarrow \varepsilon_1 = m\frac{g}{2}\Psi_1^2$$

The projection leads to the term m g $\Psi_1^2/2$, a non-linear effect emphasized by Graizer (2005, 2006). This term corresponds to the static effect of the tilt on the vertical sensor.

Passage from tilt Ψ_1 to tilt Ψ_2 when the vertical axis is taken as the reference frame

As the tilt Ψ_1 produces a deficit of weight noted ε_1, another additional tilt Ψ_2 generates another deficit $\varepsilon_2 = m\,g\,\Psi_2^2/2$, leading to a global deficit.

$$\varepsilon = \varepsilon_2 - \varepsilon_1 = m\frac{g}{2}(\Psi_2^2 - \Psi_1^2) = m\frac{g}{2}\Psi_{tilt}(\Psi_2 + \Psi_1) \approx mg\Psi_1\Psi_{tilt}$$

The equation of motion becomes

$$\ddot{V}_z + 2\beta\omega_0\dot{V}_z + \omega_0^2 V_z = -\ddot{U}_z + g\Psi_1\Psi_{tilt} - d\ddot{\Psi}_{tilt} \qquad (2)$$

where the differential tilt shows up through its value and its acceleration, an illustration of the non-linear vertical response to a tilt effect.

Instrumental response of inertial sensors

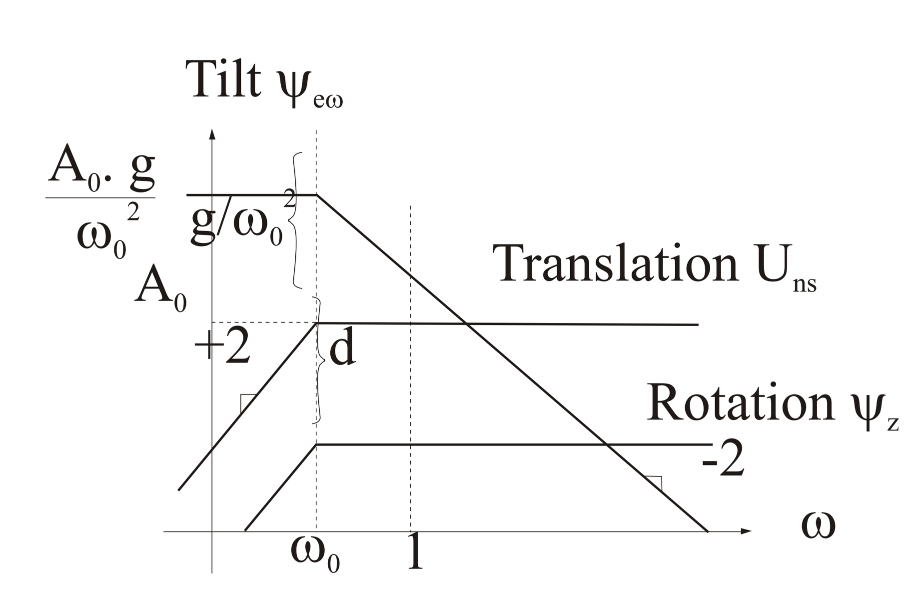

Fig4 : Instrument response for both horizontal translation and two rotations

Horizontal sensors: one may consider different contributions to the instrumental recorded signal Y_{ns}. The first contribution comes from the rotation Ψ_{ew} around the horizontal axis after a Laplace transformation of the equation (1).

$$Y_{ns}/\Psi_{ew} = \frac{g}{(s^2 + 2.s.\beta.\omega_0 + \omega_0^2)}$$

Acceleration U_{ns} and rotation Ψ_z act as a second-order high-pass filter with different sensitivity through the relation

$$Y_{ns}/U_{ns} = Y_{ns}/(d\Psi_z) = \frac{-s^2}{(s^2 + 2s\beta\omega_0 + \omega_0^2)}$$

In order to distinguish contributions between these three degrees of freedom for a single instrument, we must perform approximations by considering specific frequency bands.

Vertical sensors: we may proceed the same way by considering different contributions to the instrumental signal V_z:

We may consider acceleration terms

$$V_z/U_z = V_z/(d\Psi_{tilt}) = \frac{-s^2}{(s^2 + 2s\beta\omega_0 + \omega_0^2)}$$

and rotational contributions

$$V_z/(\Psi_1\Psi_{tilt}) = \frac{g}{(s^2 + 2s\beta\omega_0 + \omega_0^2)}$$

We must underline that vertical sensors are quite sensitive to the amplitude of tilts through a non-linear term while horizontal sensors are more linear. Sometimes, temporal filtering makes the separation possible with a leading contribution of one of degrees of freedom.

In order to have only negligeable contribution of tilt, one must install the seismometer using a strict protocol for matching the vertical axis and the translation axis.

The hinged seismometer

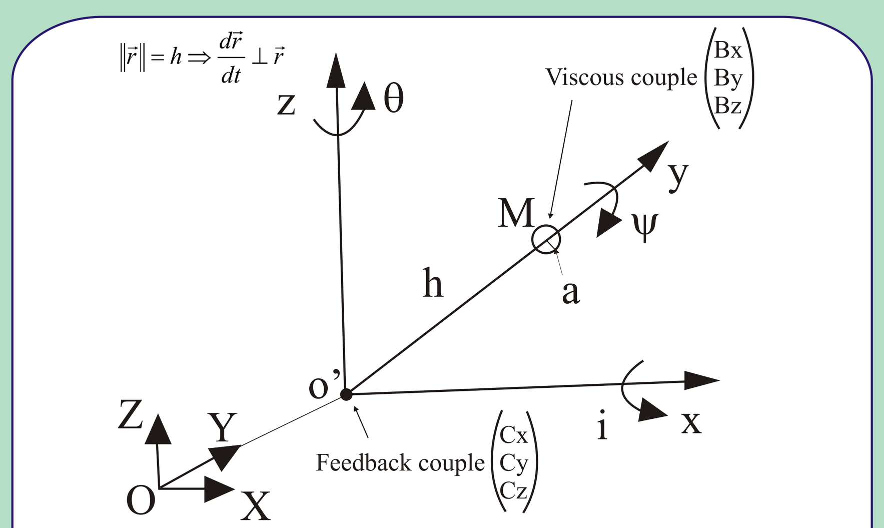

Fig 3: Coordinate frames for the hinged seismometer

Inertial moments

$$I = I_x = I_z = Mh^2$$

$$J = I_y = \frac{5}{2}Ma^2 << I$$

Three degrees of freedom

Horizontal rotation through the mechanical apparatus of the garden-gate suspension is specified by the degree of freedom Θ, angle of rotation around the vertical z axis. The instrument may have a quasi-static frame rotation θ_0 and a small vibration rotation θ.

Vertical motion through the mechanical apparatus of the “La Coste” suspension is specified by the degree of freedom i, the angle of rotation around the horizontal x axis. The instrument may have a quasi-static frame rotation i_0 and a small vibration rotation i.

Last degree of freedom Ψ is the angle related to rotation around the horizontal y axis which is essentially the quasi-static frame rotation ψ_0. Vibrations are negligeable since inertial moment is quite small along this axis.

Coupled differential system for small angles θ, i

$$\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} \ddot{\theta} \\ \ddot{i} \end{pmatrix} + \begin{pmatrix} B_z & 0 \\ 0 & B_x \end{pmatrix}\begin{pmatrix} \dot{\theta} \\ \dot{i} \end{pmatrix} + \begin{pmatrix} M\ddot{u}_y h + C_z & 0 \\ 0 & M\ddot{u}_y h + C_x \end{pmatrix}\begin{pmatrix} \theta \\ i \end{pmatrix} + \begin{pmatrix} 0 & Mgh\psi_0\theta \\ Mgh\psi_0 i & 0 \end{pmatrix}\begin{pmatrix} \theta \\ i \end{pmatrix} = \begin{pmatrix} -Mgh\psi_0 + M\ddot{u}_x h \\ -Mgh - M\ddot{u}_z h \end{pmatrix}$$

Uncoupled differential system for small angles θ, i

$$\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} \ddot{\theta} \\ \ddot{i} \end{pmatrix} + \begin{pmatrix} B_z & 0 \\ 0 & B_x \end{pmatrix}\begin{pmatrix} \dot{\theta} \\ \dot{i} \end{pmatrix} + \begin{pmatrix} M\ddot{u}_y h + C_z + Mgh\psi_0 i_0 & 0 \\ 0 & M\ddot{u}_y h + C_x + Mgh\psi_0\theta_0 \end{pmatrix}\begin{pmatrix} \theta \\ i \end{pmatrix} = \begin{pmatrix} -Mgh\psi_0 + M\ddot{u}_x h \\ -Mgh - M\ddot{u}_z h \end{pmatrix}$$

Analysis of differences between solutions of these two systems should be performed in order to check frame rotation influences in vibration quantities. Numerical integration is necessary for such purpose.

Rotations Around the Vertical Axis

For very long-period signals, horizontal components of the same broadband station present sometimes quite similar signals of background noise. In other words, the two horizontal components have nearly same phase and amplitude, as if the instrument records the same background noise on both traces. Coherency between these signals is sometimes higher than 95% in the band of period (2000s-50s).

Particule motion pattern in the horizontal plane shows a wave polarization towards the N45° degree of azimuth (Fig 5). If sensors record only translation motion, this correlation is unexpected: the background noise cannot be permanently polarized along a specific axes in a given station. Rotation around the vertical axis on horizontal components may act similarly on both horizontal components when considering the instrument design, leading to this expected correlation. Two perpendicular garden-gate sensors will record similar motion related to this rotation.

Figure 5 represents two examples of this correlation between horizontal components at two GEOSCOPE network broadband stations equiped with Wielandt-Streckeisen's seismometers STS-1 : NOUC, New Caledonia and PPT, Papeete, Tahiti.

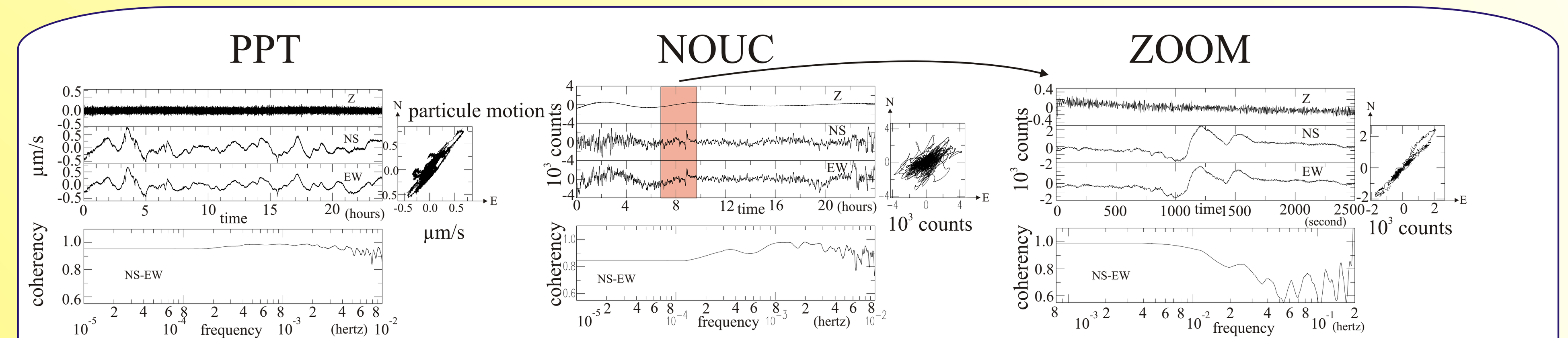

Fig5: Observation of a strong correlation between signals recorded on horizontal components at two GEOSCOPE stations: rotation around the vertical axis might be an explanation if sensors sensitive to rotations.

Co-Seismic Displacements and Tilts in the Near Field

The double integration of strong motion acceleration recordings rarely leads to acceptable results. In the near field, significant rotations disturb accelerometer recordings and must be considered when performing the integration procedure. These co-seismic tilts must be estimated in the near field,

Tilts

Slopes of the horizontal velocity traces include the static part of the tilt. Measured amplitude and azimuth of this estimation of the static tilt lead to rather disappointing results as experienced on records of the accelerometer network of Taiwan after the Chi-Chi Earthquake (1999, Mw=7.6). Tilt amplitudes do not exhibit spatial pattern with distance decrease for example from the fault zone. Azimuths of tilts are dispersed as well without any direction or specific pattern.

These dispersions have been observed on other instrumental sites and may question us on what is really occuring at the recorded site when seismic waves travel. Either the slopes of the horizontal velocity comes from combined different effects (tilts, time variations of gravity, other effects...) or local site amplification of tilts or even an instrumental perturbation essentially in the coupling between the instrument and the ground.

Co-Seismic Displacement

As shown on the figure 6, the detection of the co-seismic tilt jump is performed on the velocity signal where appears a strong drift. We may remove the drift before the integration leading to a well-defined displacement quantity. This static value is over-estimated: more than 14 meters in the TCU068 station (located at the northern end of the fault of Chelungpu) whereas close GPS measurements give only 10 meters. Although the double integration is still a very sensitive procedure to small errors, we may mention that the relation between the rotational signal and the synthetic function for the slope estimation should be investigated. In other words, seismologists should correct translational recordings from rotations estimated by other means as specific rotational sensors. This will give us better constrains in ground motion estimation in the near field and consequently a better description of finite seismic sources.

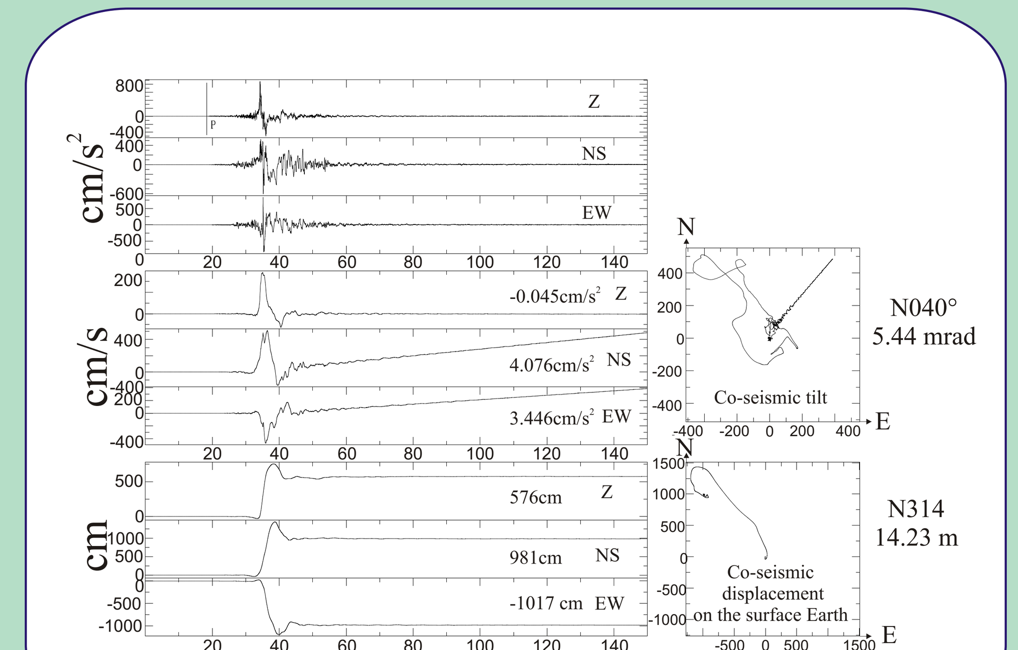

Fig6 : Double-time integration (left panel) and horizontal particule motion (right panel)

CONCLUSION

Well-known inertial sensors are also sensitive to rotational motions : rotation around an horizontal axis (tilt) and rotation around vertical axis.

- Seismic source generates significant rotations which disturb the recordings in the near field.

- Measurement of the co-seismic displacement becomes quite accurate if one could correct acceleration records of disturbing tilts. Additional sensors are required for such purpose.

- Measurement of the co-seismic tilt (amplitude and azimuth) from horizontal components do not yet provide convincing results. Maybe local effects are prevalent.

- For long period seismic background noise, the rotation around the vertical axis may affect both horizontal signals: it is quite noticeable at different stations of the Geoscope network. Specific consideration should be taken when using seismic noise by proper design of instruments.

References

- Rodgers P. W., The response of the horizontal pendulum seismometer to Rayleigh and Love waves, tilt, and free oscillations of the Earth, Bull. Seism. Soc. Am., 58, 5, (1968), 1384-1406.

- Bradner H. and M. Reichle, Some methods for determining acceleration and tilt by use of pendulums and accelerometers. Bull. Seism. Soc. Am., 63, 1, (1973), 1-7.

- Graizer V. M., Effect of tilt on strong motion data processing. Soil Dynamics and Earthquake Engineering, 25,(2005), 197-204.

- Graizer V. M., Equation of pendulum motion including rotations and its implications to the strong-ground motion. in Earthquake Source asymmetry, Structural Media and Rotation Effects, (2006), 471-485.