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Stochastic modeling of rocking seismic ground motion and respective structural load modeling

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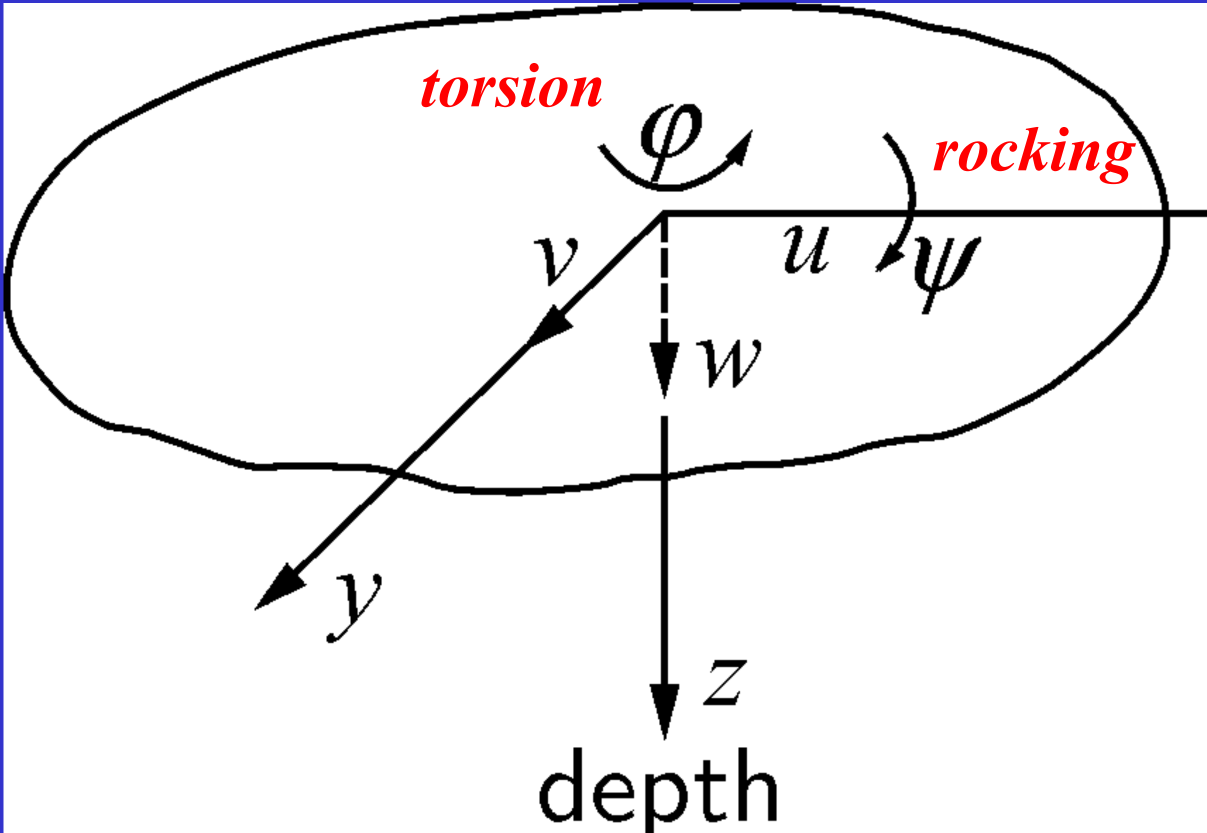
A system of principal[1] axes on the ground surface



$$\varphi = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\psi = \frac{\partial w(x,y,t)}{\partial x}$$

components *u(t), v(t), w(t)* are uncorrelated

φ – horizontal rotation about *z* axis (torsion)

ψ – vertical rotation about *y* axis (rocking)

[1] Penzien & Watabe Characteristics of 3-D earthquake ground motion, *Earthquake Eng. & Structural Dynamics*, **3,** 1975

Stochastic model of seismic ground motion

Classic Fourier-Stieltjes representation of stationary random processes:

$$\ddot{u}(t) = \int_{-\infty}^{\infty} e^{i\omega t}\, d\hat{\ddot{u}}(\omega)$$

the dashed symbol $\hat{\ddot{u}}(\omega)$ it is random function in frequency domain with orthogonal increments:

$$\left\langle d\hat{\ddot{u}}(\omega_1)\, d\hat{\ddot{u}}^*(\omega_2)\right\rangle = \begin{cases} \left\langle \left| d\hat{\ddot{u}}(\omega)\right|^2 \right\rangle = S_{\ddot{u}}(\omega)\mathrm{d}\omega \quad \text{for} \quad \omega_1 = \omega_2 = \omega \\ 0 \quad \text{for} \quad \omega_1 \neq \omega_2 \end{cases}$$

A typical model of ground acceleration it is the Kanai-Tajimi spectrum :

$$S(\omega) = \frac{\omega_g^4 + (2\xi_g \omega_g \omega)^2}{(\omega_g^2 - \omega^2)^2 + (2\xi_g \omega_g \omega)^2} S_0$$

in which the ω_g and ξ_g are parameters representing local site conditions:

Method of evolutionary spectrum

In **1967 Priestley** proposed an evolutionary representation for non-stationary random processes in following form

$$\ddot{u}(t) = \int_{-\infty}^{\infty} A_X(t,\omega) e^{i\omega t} d\hat{\ddot{u}}(\omega)$$

It is the modulating function $A_X(t,\omega)$ which controls the time-dependence (non-stationarity) of the accelearion process

the dashed symbol $\hat{\ddot{u}}(\omega)$ it is, as previously, random function in frequency domain with orthogonal increments:

$$\left\langle d\hat{\ddot{u}}(\omega_1) d\hat{\ddot{u}}^*(\omega_2) \right\rangle = \begin{cases} \left\langle \left| d\hat{\ddot{u}}(\omega) \right|^2 \right\rangle = S_{\ddot{u}}(\omega) \mathrm{d}\omega & \text{for} \quad \omega_1 = \omega_2 = \omega \\ 0 \quad \text{for} \quad \omega_1 \neq \omega_2 \end{cases}$$

Evolutionary Kanai-Tajimi earthquake model

In **1987 Lin & Yong** proposed a generalization of the familiar Kanai-Tajimi model of seismic ground motion to account for non-stationarity of the excitation process
The evolutionary spectral density as proposed by Lin & Yong takes form:

$$S_{\ddot{u}}(t,\omega) = |M(t,\omega)|^2 \; S_0$$

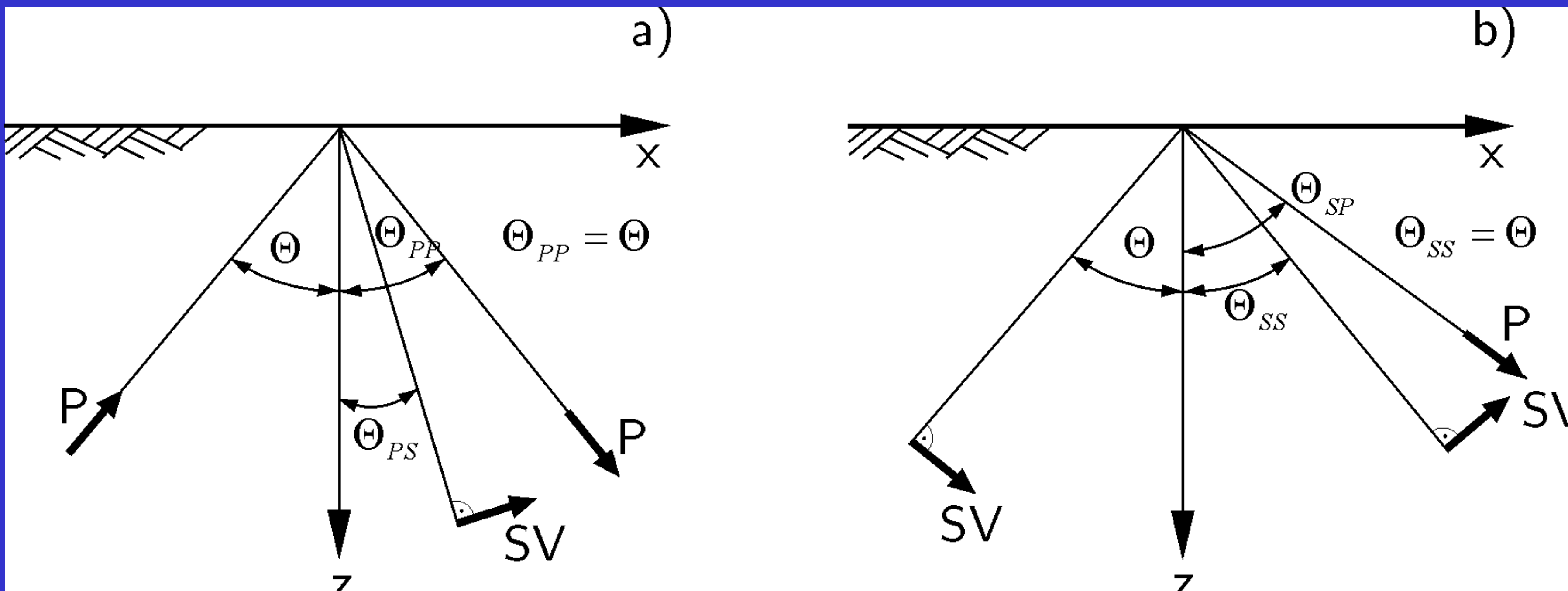
$$with \quad M(t,\omega) = \int_0^t h(\tau)\sqrt{\lambda(t-\tau)}e^{i\omega\tau}d\tau$$

$$h(t) = \omega_g \exp[-\xi_g^2\omega_g t]\left\{\frac{1-2\xi_g^2}{\sqrt{1-\xi_g^2}}\sin(\omega_{gd}t) + 2\xi_g\cos(\omega_{gd}t)\right\}$$

$$\omega_{gd} = \omega_g\sqrt{1-\xi_g^2}$$

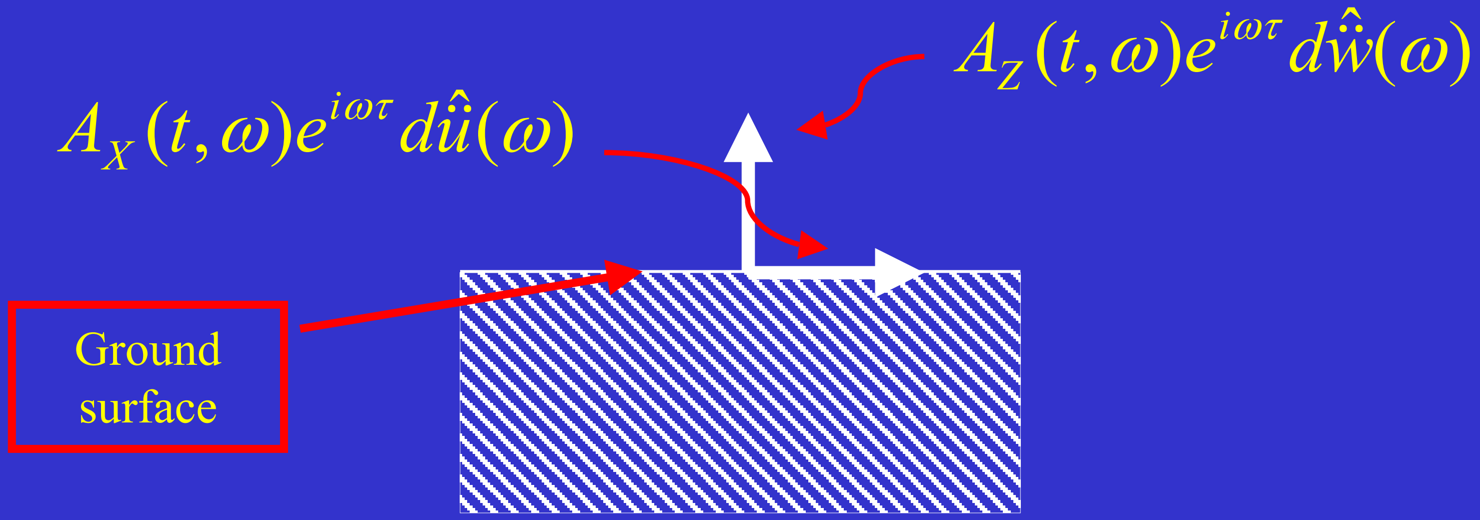
Trifunac M.D. A note on rotational components ofearthquake motions on ground surface for incident body waves, Soil *Dynamics & Earthquake Engineering*, ***1***, *1982, pp.11-19.*

Decomposition of **body waves**



Assumption: the same incidence angle *Θ* for both *P & SV* waves

Soil not stratified



Horizontal excitations as contribution of body P and SV waves

$$d\ddot{u} = d\ddot{u}_P + d\ddot{u}_S$$

Vertical excitations as contribution of body P and SV waves

$$d\ddot{w} = d\ddot{w}_P + d\ddot{w}_S$$

frequency band: $(\omega;\ \omega+d\omega)$

Substituting the Priestley'a spectral represenation

$$\begin{cases} A_X(t,\omega)e^{i\omega\tau}d\hat{\hat{u}}(\omega) = U_p A_P(t,\omega)e^{i\omega\tau}d\hat{\hat{\Phi}}_p(\omega) \\ \quad + U_s A_S(t,\omega)e^{i\omega\tau}d\hat{\hat{\Phi}}_s(\omega) \\ A_Z(t,\omega)e^{i\omega\tau}d\hat{\hat{w}}(\omega) = W_p A_P(t,\omega)e^{i\omega\tau}d\hat{\hat{\Phi}}_p(\omega) \\ \quad + W_s A_S(t,\omega)e^{i\omega\tau}d\hat{\hat{\Phi}}_s(\omega) \end{cases}$$

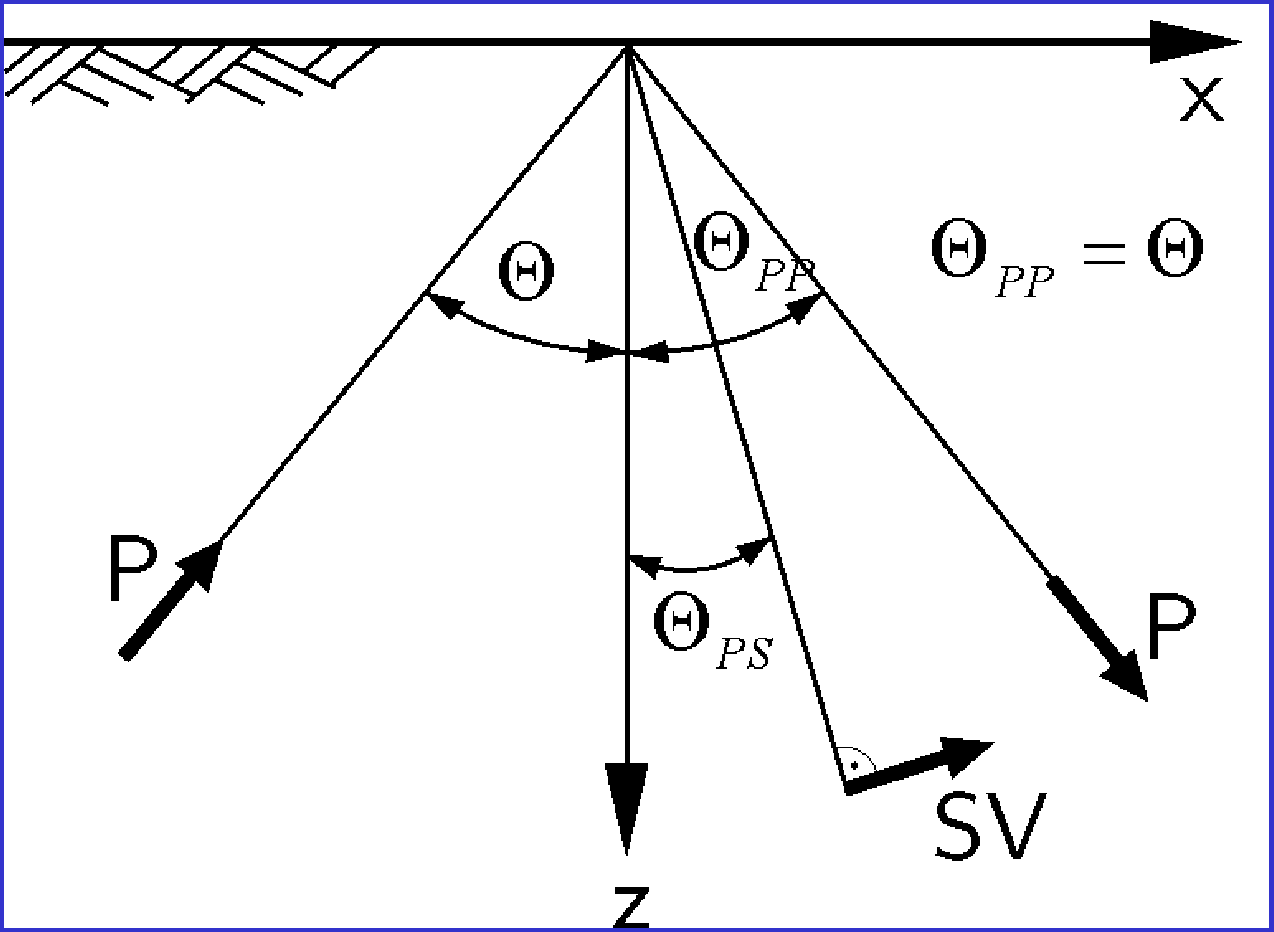
U_P, U_S – horizontal participation coefficients

W_P, W_S - vertical participation coefficients

Coefficients U_P & W_P

$$U_p = (1 + P_p)\sin(\Theta) + P_s \cos(\Theta_{ps})$$

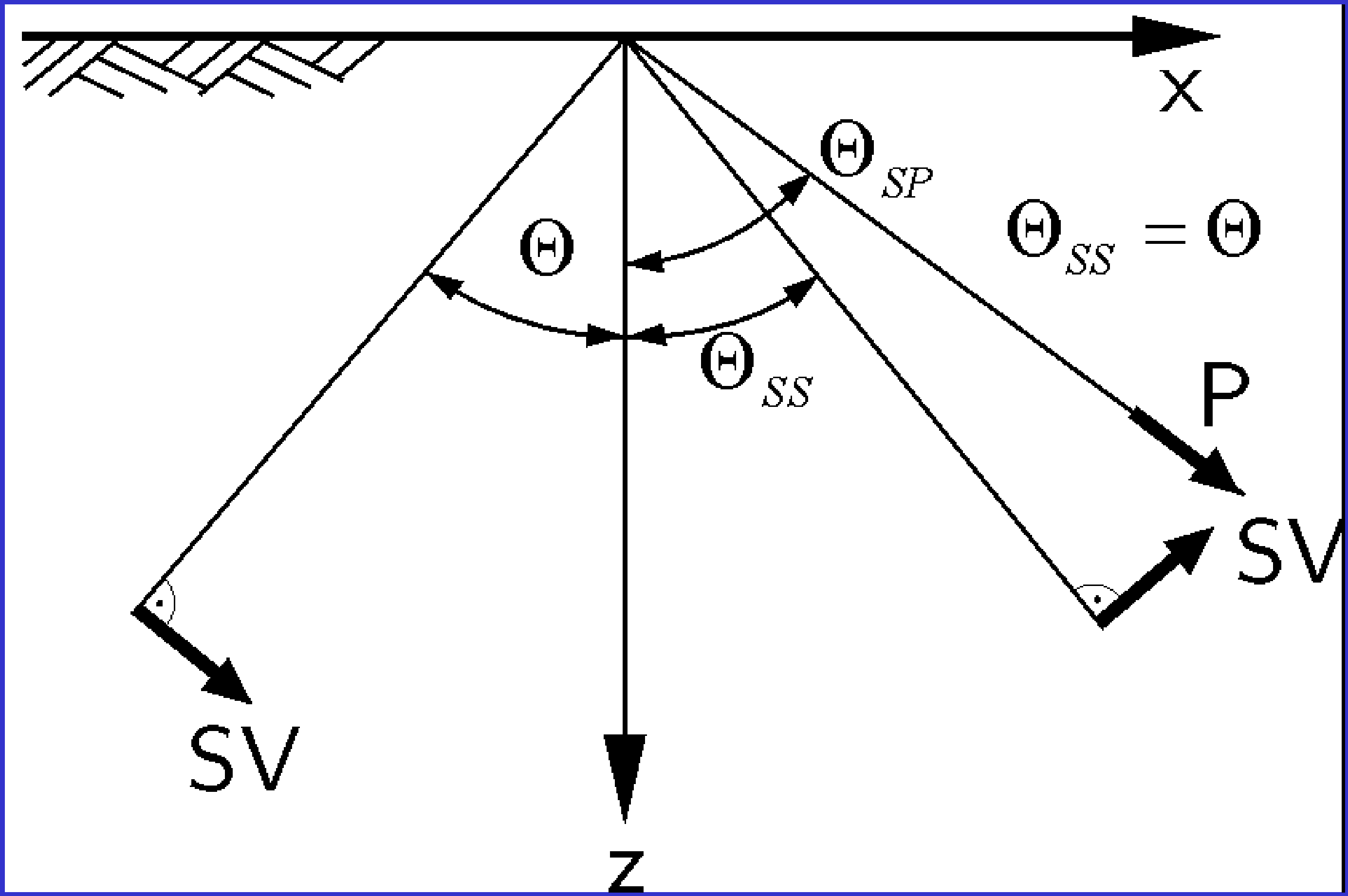
$$W_p = (P_p - 1)\cos(\Theta) + P_s \sin(\Theta_{ps})$$



Coefficients U_S & W_S

$$U_s = (1 + S_s)\cos(\Theta) + S_p \sin(\Theta_{sp})$$

$$W_s = (1 - S_s)\sin(\Theta) - S_p \cos(\Theta_{sp})$$



The coefficients P_P, P_S & S_P, S_S are reflection coefficients of body waves at the free surface

$$P_p = \frac{-\left(\frac{1}{c_s^2} - 2p^2\right)^2 + 4p^2 \frac{\cos(\Theta)}{c_p} \frac{\cos(\Theta_{ps})}{c_s}}{\left(\frac{1}{c_s^2} - 2p^2\right)^2 + 4p^2 \frac{\cos(\Theta)}{c_p} \frac{\cos(\Theta_{ps})}{c_s}} \qquad S_p = \frac{4\frac{c_p}{c_p} p \frac{\cos(\Theta)}{c_s} \left(\frac{1}{c_s^2} - 2p^2\right)}{\left(\frac{1}{c_s^2} - 2p^2\right)^2 + 4p^2 \frac{\cos(\Theta_{sp})}{c_p} \frac{\cos(\Theta)}{c_s}}$$

$$P_s = \frac{4\frac{c_p}{c_s} p \frac{\cos(\Theta)}{c_p} \left(\frac{1}{c_s^2} - 2p^2\right)}{\left(\frac{1}{c_s^2} - 2p^2\right)^2 + 4p^2 \frac{\cos(\Theta)}{c_p} \frac{\cos(\Theta_{ps})}{c_s}} \qquad S_s = \frac{\left(\frac{1}{c_s^2} - 2p^2\right)^2 - 4p^2 \frac{\cos(\Theta_{sp})}{c_p} \frac{\cos(\Theta)}{c_s}}{\left(\frac{1}{c_s^2} - 2p^2\right)^2 + 4p^2 \frac{\cos(\Theta_{sp})}{c_p} \frac{\cos(\Theta)}{c_s}}$$

Achenbach, *Wave propagation in elastic solids*, North-Holland, Amsterdam 1973
Aki, Richards, *Quantitatove Seismology*, W.H. Freeman, San Francisco 1980,

$$\sin(\Theta)/\sin(\Theta_{ps}) = c_p / c_s = S \qquad \sin(\Theta)/\sin(\Theta_{sp}) = c_s / c_p$$

C_P= propagation velocity of P waves

C_S= propagation velocity of S waves

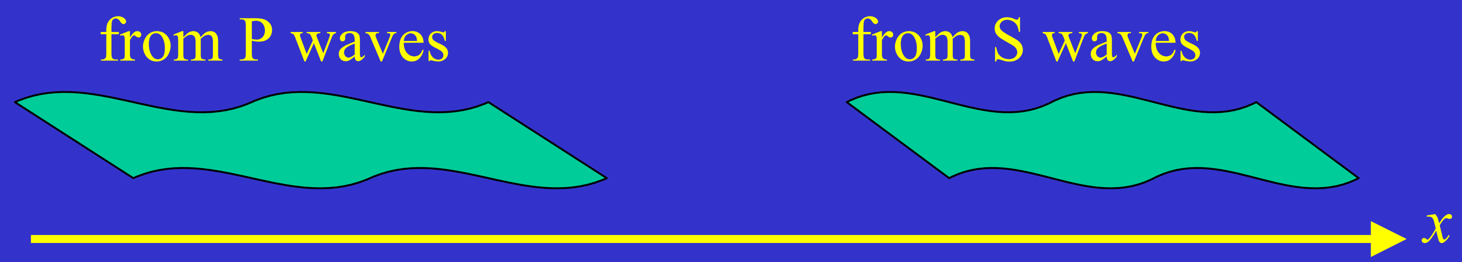
p=sin(Θ)/c_P = horizontal slowness of incident *P* waves (inverse of velocity)

$$\begin{cases} A_X(t,\omega)e^{i\omega\tau}d\hat{\hat{u}}(\omega) = U_p A_P(t,\omega)e^{i\omega\tau}d\hat{\hat{\Phi}}_p(\omega) \\ \qquad + U_s A_S(t,\omega)e^{i\omega\tau}d\hat{\hat{\Phi}}_s(\omega) \\ A_Z(t,\omega)e^{i\omega\tau}d\hat{\hat{w}}(\omega) = W_p A_P(t,\omega)e^{i\omega\tau}d\hat{\hat{\Phi}}_p(\omega) \\ \qquad + W_s A_S(t,\omega)e^{i\omega\tau}d\hat{\hat{\Phi}}_s(\omega) \end{cases}$$

solving the initial system of equations for A_P and A_S gives:

$$\begin{cases} A_P(t,\omega)e^{i\omega t}d\hat{\hat{\Phi}}_p(\omega) = \dfrac{W_s}{D} A_X(t,\omega)e^{i\omega t}d\hat{\hat{u}}(\omega) \\ \qquad - \dfrac{Us}{D} A_Z(t,\omega)e^{i\omega t}d\hat{\hat{w}}(\omega) \\ A_S(t,\omega)e^{i\omega t}d\hat{\hat{\Phi}}_s(\omega) = \dfrac{U_p}{D} A_Z(t,\omega)e^{i\omega t}d\hat{\hat{w}}(\omega) \\ \qquad - \dfrac{W_s}{D} A_X(t,\omega)e^{i\omega t}d\hat{\hat{u}}(\omega) \end{cases}$$

The vertical acceleration in the frequency band (ω; ω+dω) is now decomposed onto two wave terms:



$$d\ddot{w}(t,\omega,x) = W_p \exp\left[i\omega \left(t - \frac{x\sin(\Theta)}{c_p} \right) \right] d\hat{\Phi}_p(\omega) + W_s \exp\left[i\omega \left(t - \frac{x\sin(\Theta)}{c_s} \right) \right] d\hat{\Phi}_s(\omega)$$

Differentiating the vertical field with respect to spatial, horizontal coordinate *x* gives the rocking component

$$d\ddot{\psi}(t,\omega,x) = \frac{\partial}{\partial x} d\ddot{w}(t,\omega,x)\Big|_{x=0}$$

Rocking acceleration

$$d\ddot{\psi}(t,\omega) = W_p A_X(t,\omega)\left(-i\omega\frac{\sin(\Theta)}{c_p}\right)e^{i\omega t}\frac{W_s}{D}d\hat{\ddot{u}}(\omega)$$

$$-W_p A_Z(t,\omega)\left(-i\omega\frac{\sin(\Theta)}{c_p}\right)e^{i\omega t}\frac{U_s}{D}d\hat{\ddot{w}}(\omega)$$

$$+W_s A_Z(t,\omega)\left(-i\omega\frac{\sin(\Theta)}{c_p}\right)e^{i\omega t}\frac{U_p}{D}d\hat{\ddot{w}}(\omega)$$

$$-W_s A_X(t,\omega)\left(-i\omega\frac{\sin(\Theta)}{c_s}\right)e^{i\omega t}\frac{W_p}{D}d\hat{\ddot{u}}(\omega)$$

Introducing new coefficients

$$W_x = \frac{W_p W_s}{D}\frac{\sin(\Theta)}{c_p} - \frac{W_p W_s}{D}\frac{\sin(\Theta)}{c_s}$$

$$W_z = \frac{U_p W_s}{D}\frac{\sin(\Theta)}{c_s} - \frac{W_p U_s}{D}\frac{\sin(\Theta)}{c_p}$$

we have

$$d\ddot{\psi}(t,\omega) = W_x(-i\omega)A_X(t,\omega)e^{i\omega t}d\hat{\ddot{u}}(\omega) + W_z(-i\omega)A_Z(t,\omega)e^{i\omega t}d\hat{\ddot{w}}(\omega)$$

After integration in the whole frequency domain we have rocking acceleration:

$$\ddot{\psi}(t) = \int_{-\infty}^{\infty} W_x(-i\omega)A_X(t,\omega)e^{i\omega t}d\hat{\ddot{u}}(\omega) + \int_{-\infty}^{\infty} W_z(-i\omega)A_Z(t,\omega)e^{i\omega t}d\hat{\ddot{w}}(\omega)$$

This equation can be used to obtain any probabilistic characterisitics of rotation

For example the mean square rotational acceleration

$$\sigma_{\ddot{\psi}}^2(t) = \int_{-\infty}^{\infty} \left|W_x\right|^2 |A_X(t,\omega)|^2 \omega^2 S_{\ddot{u}}(\omega) d\omega + 2\int_{-\infty}^{\infty} W_x W_z^* A_X(t,\omega) A_Z^*(t,\omega) \omega^2 S_{\ddot{u}\ddot{w}}(\omega) d\omega$$

$$+ \int_{-\infty}^{\infty} \left|W_z\right|^2 |A_Z(t,\omega)|^2 \omega^2 S_{\ddot{w}}(\omega) d\omega$$

Introducing the Priestely's formulae for evolutionary spectrum

$$\sigma_{\ddot{\psi}}^2(t) = \int_{-\infty}^{\infty} \left|W_x\right|^2 \omega^2 S_{\ddot{u}}(t,\omega) d\omega + 2\int_{-\infty}^{\infty} W_x W_z^* \omega^2 S_{\ddot{u}\ddot{w}}(t,\omega) d\omega + \int_{-\infty}^{\infty} \left|W_z\right|^2 \omega^2 S_{\ddot{w}}(t,\omega) d\omega$$

The integrand in this equation it is the evolutionary spectrum

$$S_{\ddot{\psi}}(t,\omega) = \left|W_x\right|^2 \omega^2 S_{\ddot{u}}(t,\omega) + 2 W_x W_z^* \omega^2 S_{\ddot{u}\ddot{w}}(t,\omega) + \left|W_z\right|^2 \omega^2 S_{\ddot{w}}(t,\omega)$$

In the stationary case:

$$S_{\ddot{\psi}}\omega) = \left|W_x\right|^2 \omega^2 S_{\ddot{u}}(\omega) + 2 W_x W_z^* \omega^2 S_{\ddot{u}\ddot{w}}(\omega) + \left|W_z\right|^2 \omega^2 S_{\ddot{w}}(\omega)$$

$$S_{\ddot{\psi}}\omega) = |W_x|^2 \omega^2 S_{\ddot{u}}(\omega) + 2W_x W_z^* \omega^2 S_{\ddot{u}\ddot{w}}(\omega) + |W_z|^2 \omega^2 S_{\ddot{w}}(\omega)$$

the spectrum of the rotational component is then a function of the first derivative of accelerations and the third derivative of translations

$$S_{\dddot{u}}(\omega) = \omega^2 S_{\ddot{u}}(\omega) = \omega^4 S_{\dot{u}}(\omega) = \omega^6 S_u(\omega)$$

Example:

c_P [m/s]	6800	5200	4500
c_S [m/s]	3000	3000	3000
$S=c_P/c_S$	2.27	1.73	1.50
Poisson modulus ν	0.38	0.25	0.10
Θ_{cr} [deg]	26.14	35.26	41.81

Modulae

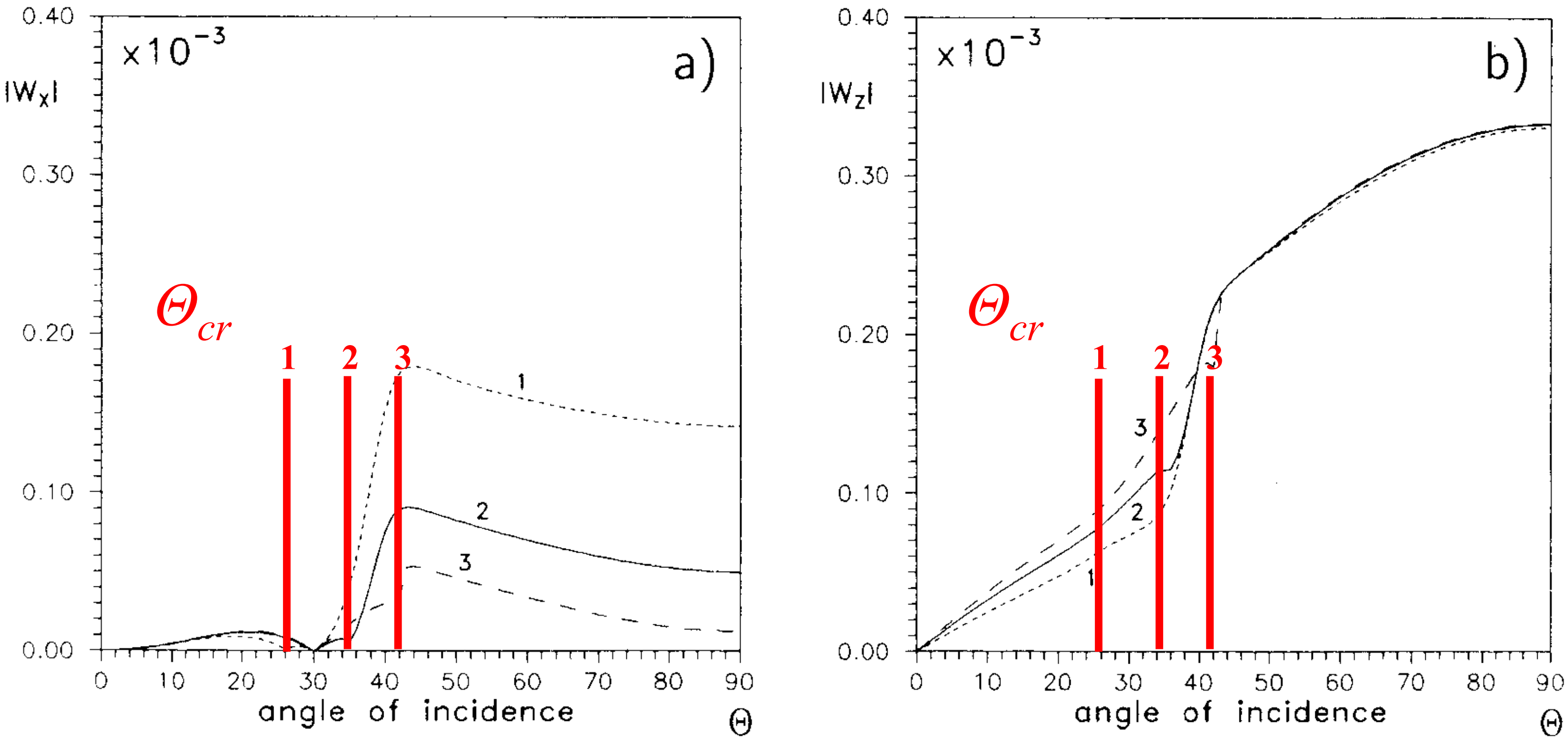
of horizontal W_X and

vertical W_Z components vs. incidence angle Θ and three values of Poisson coefficient ν

1 $\nu=0.38$ $\Theta_{cr}=26.14°$

2 $\nu=0.25$ $\Theta_{cr}=35.26°$

3 $\nu=0.10$ $\Theta_{cr}=41.81°$



for $\Theta<\Theta_{cr}$ $W_X, W_Z \in \mathcal{R}$ and $W_X<0$, $W_Z>0$

for $\Theta>\Theta_{cr}$ $W_X, W_Z \in C$

Problem of correlation between horizontal and vertical components

Applying Penzien & Watabe (1975) assumption of no-correlation we may omitt the cross spectrum term:

$$S_{\ddot{\psi}}(t,\omega) = \left|W_x\right|^2 \omega^2 S_{\ddot{u}}(t,\omega) \; + 2W_x W_z^* \omega^2 S_{\ddot{u}\ddot{w}}(t,\omega) + \left|W_z\right|^2 \omega^2 S_{\ddot{w}}(t,\omega)$$

Assuming also the same spectrum for horizontal & vertical component but with different intensity factors such that: $S_{\ddot{w}}(t,\omega) = \eta^2 S_{\ddot{u}}(\omega)$

For uncorrelated hor-ver motions we have following formula :

$$S_{\ddot{\psi}}(t,\omega) = (|W_X|^2 + \eta^2 |W_Z|^2)\omega^2 S_{\ddot{u}}(t,\omega)$$

And on the opposite side for total (100%) correlation of hor-ver motions :

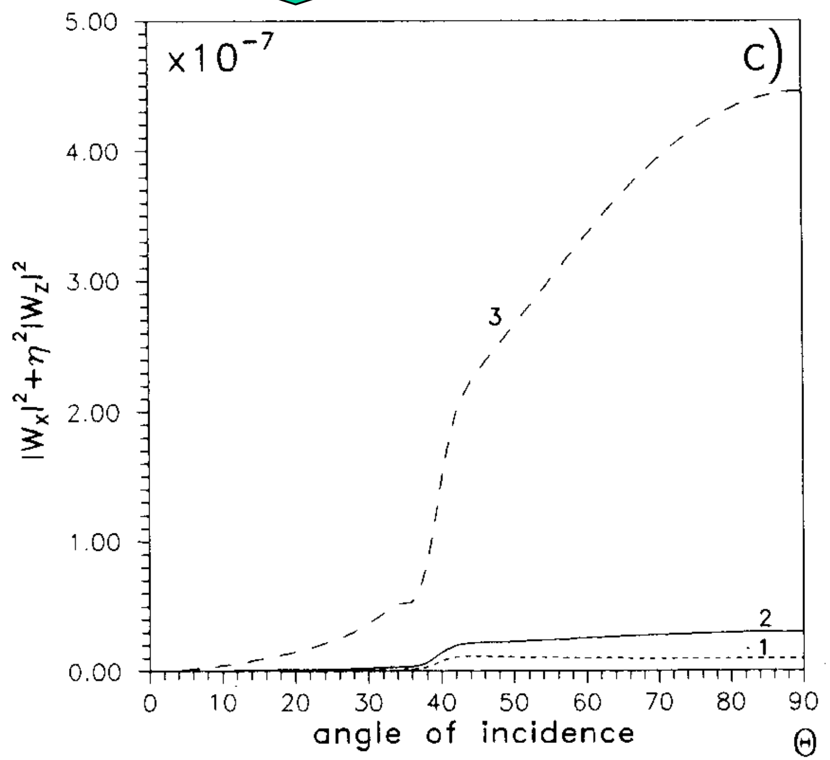
$$S_{\ddot{\psi}}(t,\omega) = (|W_X|^2 + 2\eta W_X W_Z^* + \eta^2 |W_Z|^2)\omega^2 S_{\ddot{u}}(t,\omega)$$

Both spectral multipliers vs. incidence angle, for stationary processes for three values of η: (1) η=0.25, (2) η=0.50, (3) η=2.0

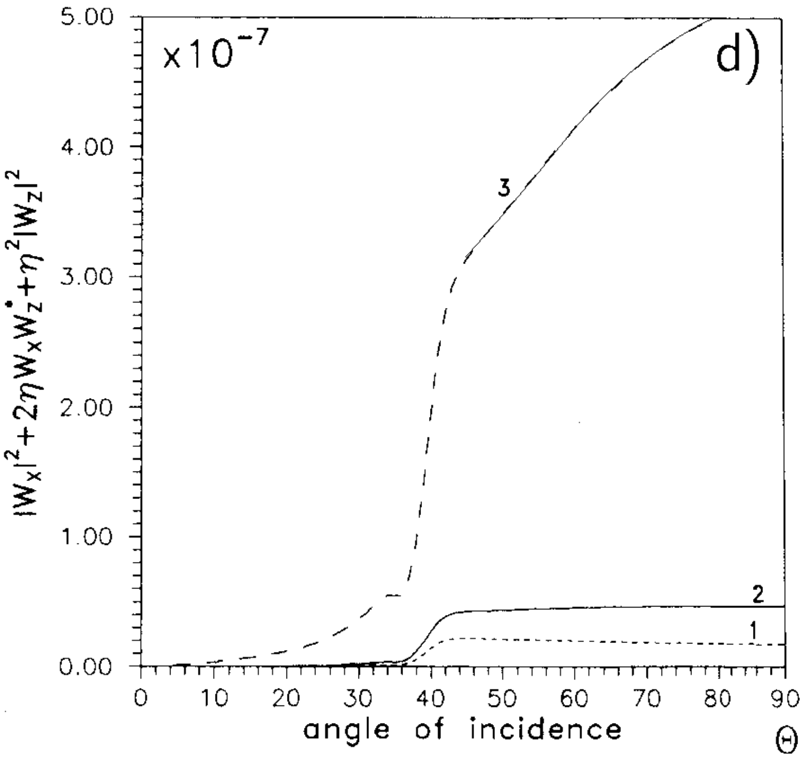
$$S_{\ddot{\psi}}(\omega) = (|W_X|^2 + \eta^2 |W_Z|^2)\omega^2 S_{\ddot{u}}(\omega)$$

$$S_{\ddot{\psi}}(\omega) = (|W_X|^2 + 2\eta W_X W_Z^* + \eta^2 |W_Z|^2)\omega^2 S_{\ddot{u}}(\omega)$$

0% correlation „hor-ver”



100% correlation „hor-ver”



translational & rotational spectrum

$$S_{\ddot{u}}(\omega) = \frac{\omega^2}{\left(1+\frac{\omega}{\omega_c}\right)^2} \exp[-\tfrac{1}{2}\kappa\omega] S_0$$

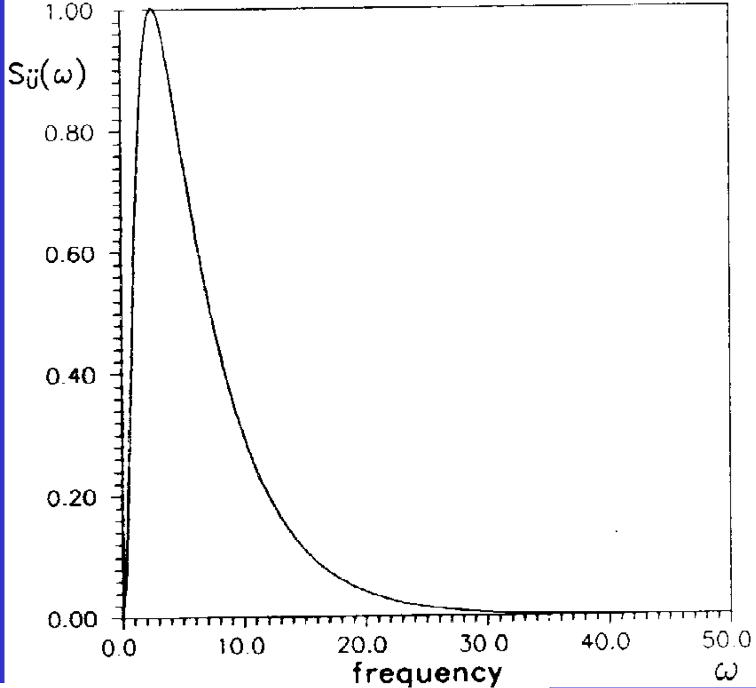
Brune spectrum; ω_c = corner frequency

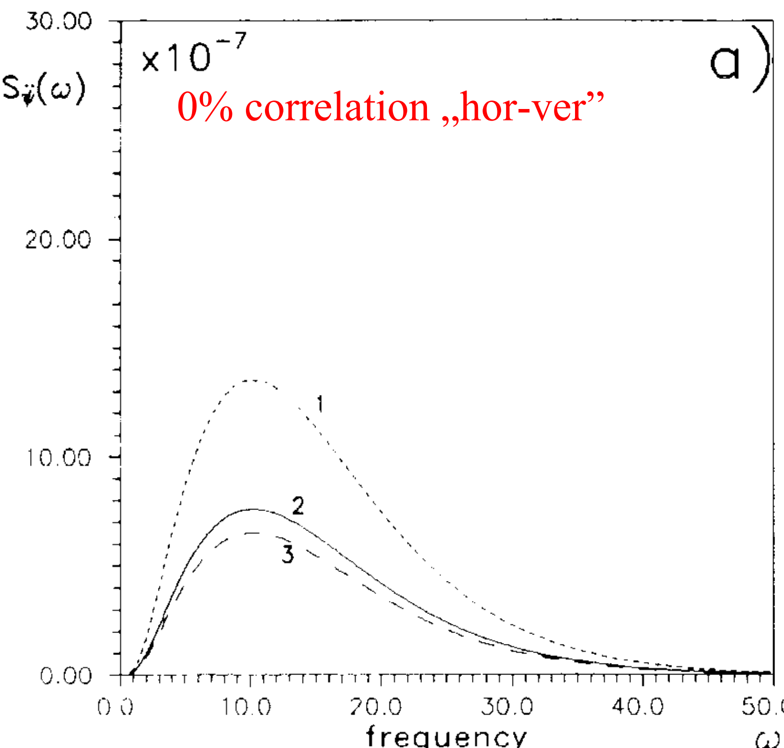
$\eta = 0.5, \Theta = 60^{\circ}$

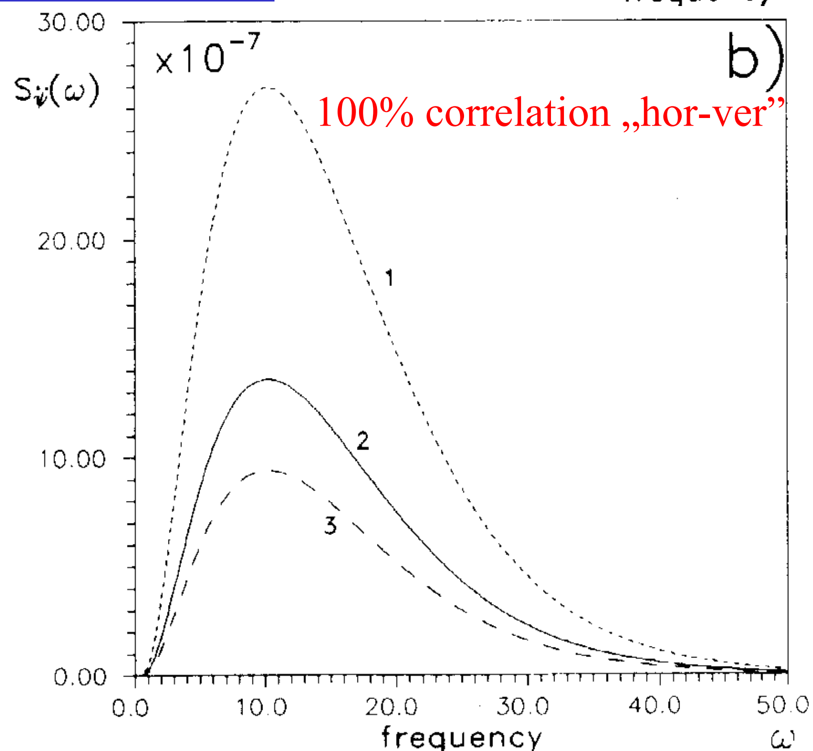
1 ν=0.38

2 ν=0.25

3 ν=0.10



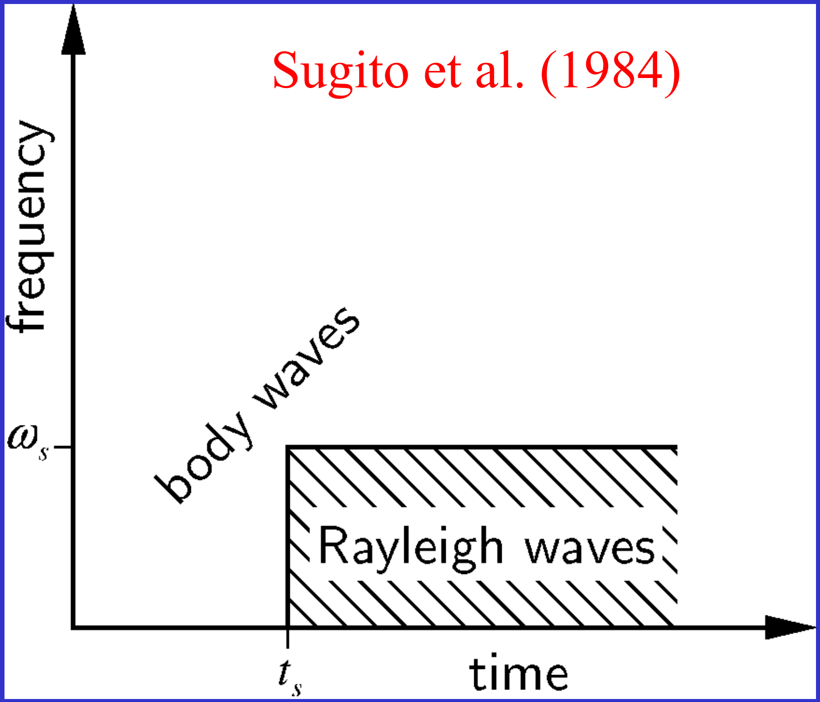




Surface waves effects

$$\ddot{u}(t) = \ddot{u}_R(t) + \ddot{u}_b(t) \quad - \text{horizontal}$$

$$\ddot{w}(t) = \ddot{w}_R(t) + \ddot{w}_b(t) \quad - \text{vertical}$$



incremental acceleration of surface waves for (ω; ω+dω):

$$d\ddot{w}_R = A_R(t,\omega)\exp\left[i\omega\left(t - \frac{x}{c_R}\right)\right]d\hat{\ddot{w}}(\omega) \qquad c_R = \begin{cases} \text{velocity of} \\ \text{Rayleigh waves} \end{cases}$$

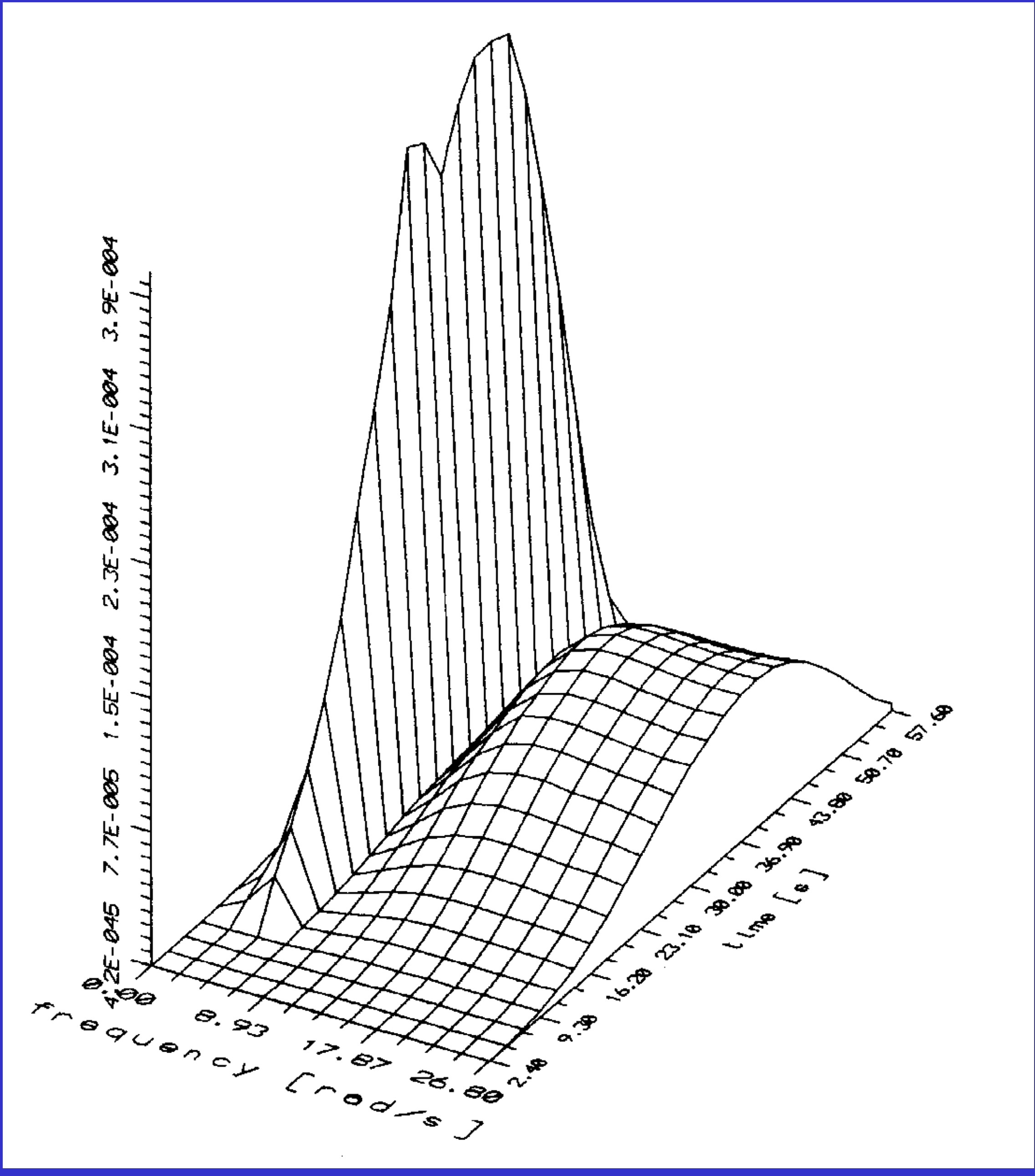
The rotation due to Rayleigh waves

$$d\ddot{\psi}_R = \frac{\partial}{\partial x} d\ddot{w}_R(t,\omega,x)\Big|_{x=0} = A_R(t,\omega)\left(-\frac{i\omega}{c_R}\right)e^{i\omega t}d\hat{\ddot{w}}(\omega)$$

Introducing $W_R = 1/c_R$ we obtain final formula
for mean square rocking from both body & surface waves

$$\sigma^2_{\ddot{\psi}}(t) = \begin{cases} 2\int_0^\infty [W_X^2\omega^2 S_{\ddot{u}}(t,\omega) + 2W_X W_Z^*\omega^2 S_{\ddot{u}\ddot{w}}(t,\omega) + W_Z^2\omega^2 S_{\ddot{w}}(t,\omega)]d\omega & \text{for } t < t_S \\ 2\int_0^{\omega_S} W_R^2\omega^2 S_{\ddot{w}}(t,\omega)d\omega + 2\int_{\omega_S}^{\omega} [W_X^2\omega^2 S_{\ddot{u}}(t,\omega) + 2W_X W_Z^*\omega^2 S_{\ddot{u}\ddot{w}}(t,\omega) + W_Z^2\omega^2 S_{\ddot{w}}(t,\omega)]d\omega & \text{for } t > t_S \end{cases}$$

Rotational evolutionary Kanai-Tajimi spectrum due to joint effect of body and surface waves:



Conclusions from the first part

The final formula for the spectral density of the rotation due to body waves:

$$S_{\ddot{\psi}}(t,\omega) = \left|W_x\right|^2 \omega^2 S_{\ddot{u}}(t,\omega) + 2W_x W_z^* \omega^2 S_{\ddot{u}\ddot{w}}(t,\omega) + \left|W_z\right|^2 \omega^2 S_{\ddot{w}}(t,\omega)$$

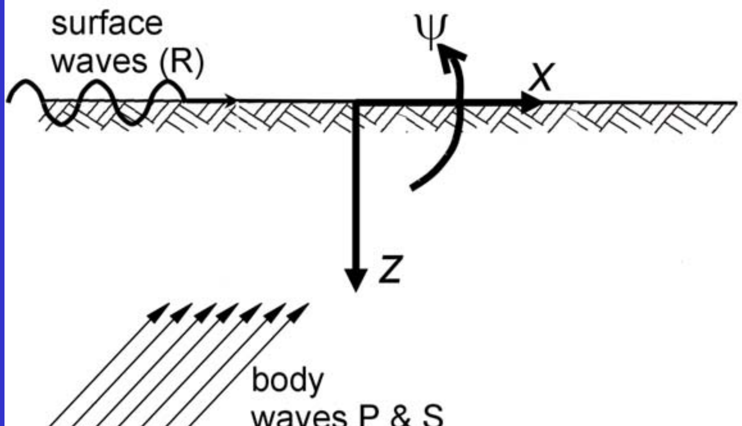
It is a function of the translational spectral densities and their cross-spectrum

$$\begin{cases} S_{\ddot{u}}(t,\omega) \\ S_{\ddot{w}}(t,\omega) \end{cases} \qquad S_{\ddot{u}\ddot{w}}(t,\omega)$$

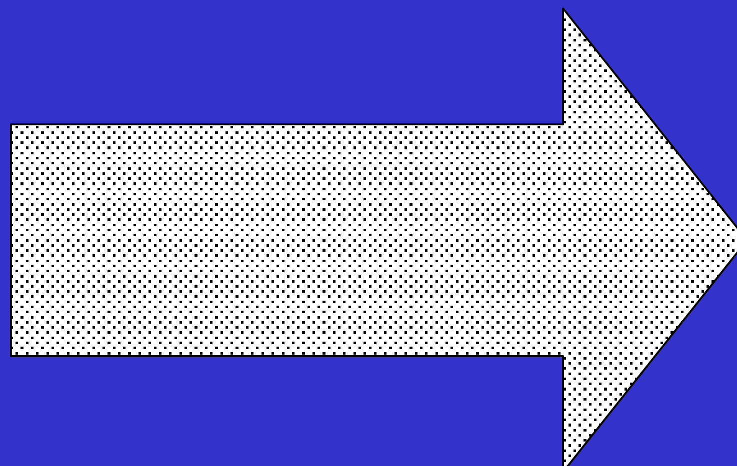
as well as wave and sub-soil parameters: $W_X, W_Z = f(\Theta, c_P, c_S)$

The ω^2 parameter shifts the rotational spectrum into higher frequencies and in the stationary case introduces direct dependence with the time derivative of the acceleration

Transforming „point" rotation onto structural load

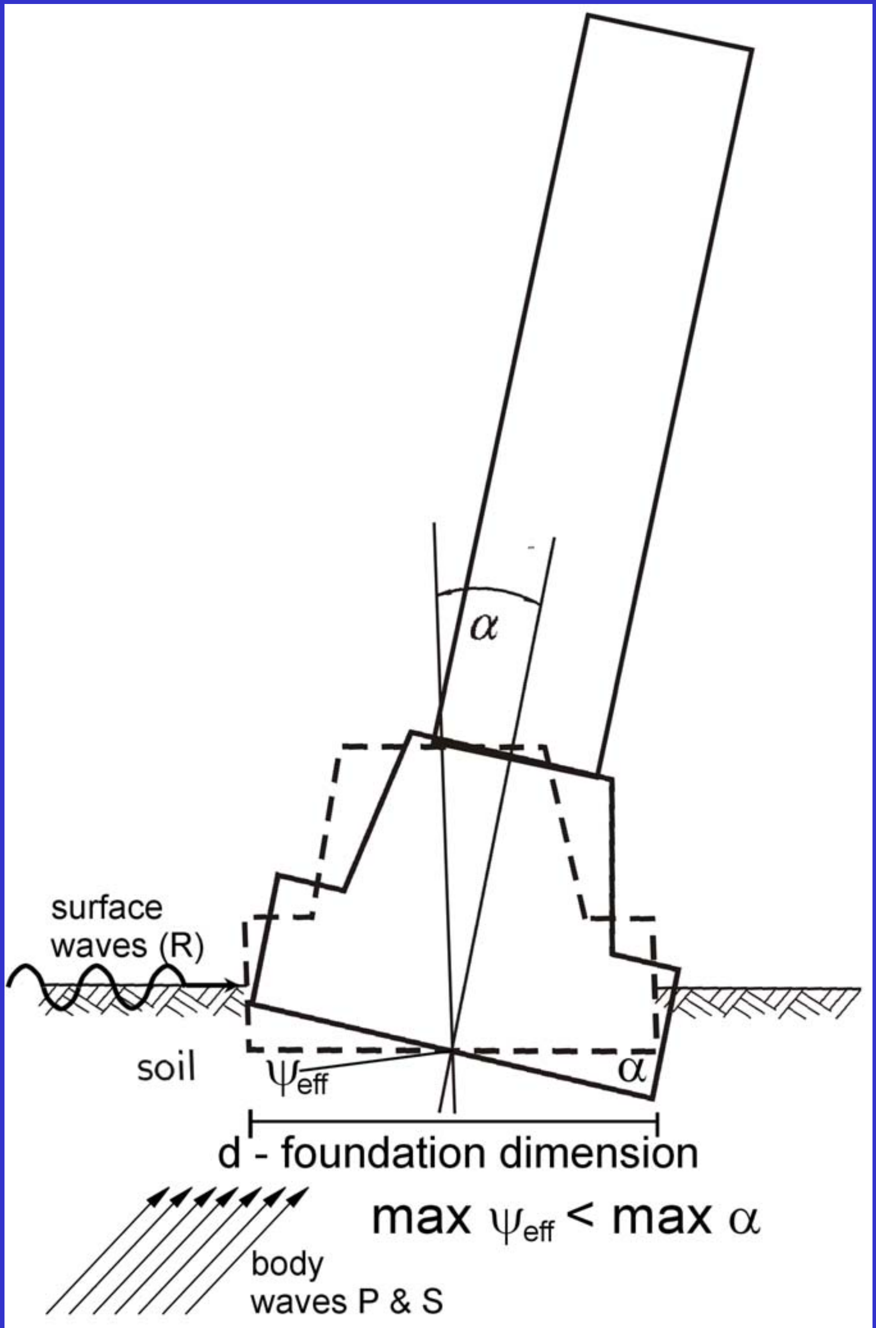


Transforming $\psi(t)$ onto $\psi_{\mathrm{eff}}(t)$

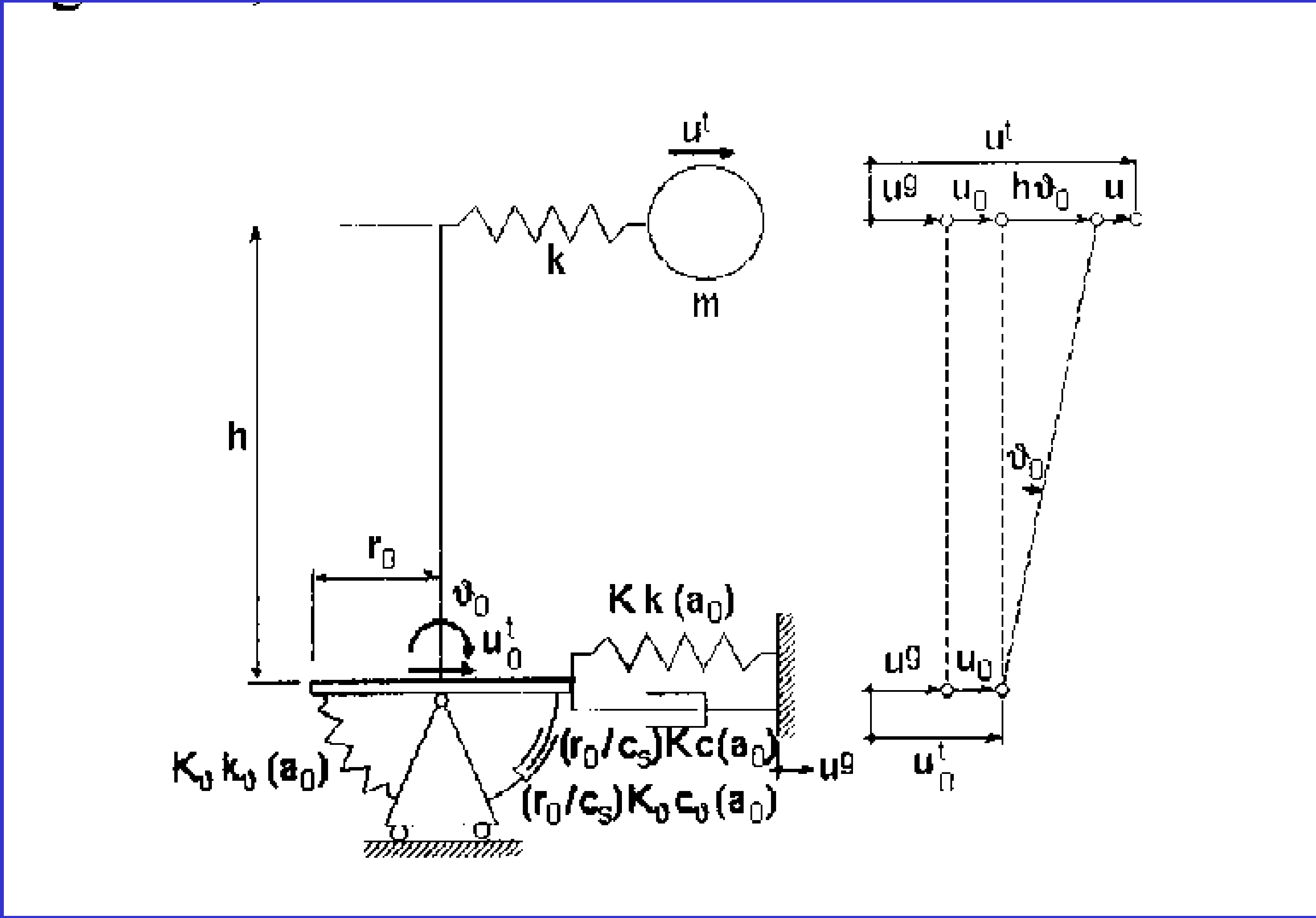
$\psi(t)$  $\psi_{\mathrm{eff}}(t)$

$\alpha(t) = \psi_{\mathrm{eff}}(t) +$  from SSI

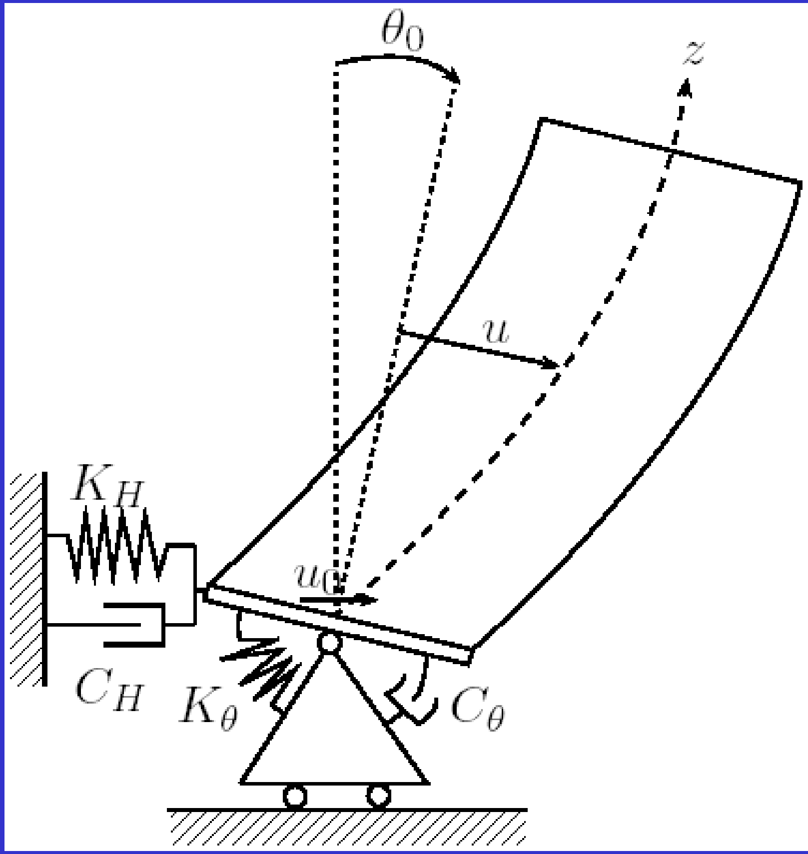
Question: How to calculate $\psi_{eff}(t)$ from $\psi(t)$



The simplest possible model of a structure under horizontal-rocking excitations



More advanced model of high-rise structure: A shear beam under horizontal-rocking excitations



Parameters for simple systems to derive from so called „cone” ground models

Translational cone	Rotational cone